Exam time: Thursday, 18 May 2006, 4:30-7:00 p.m.

Exam location: 110 Budig

Material to be covered:

1.1-1.3, 1.5-1.7
2.1-2.9
3.1-3.8
4.1-4.3, 4.5-4.6, 4.9
5.1-5.7, 5.10
6.1-6.5, 6.7

The structure of the final exam will be similar to the midterm exam. NO CALCULATORS are allowed for the exam. No books or notes are allowed. The final exam will be comprehensive.

Some sample problems are given for the material after the midterm exam. For sample problems for the prior material, consult the sample problems for the midterm and the midterm exam.
Sample Problems

1. Two nonnegative numbers, whose sum is 1 such that the sum of the square of one and twice the square of the other is a minimum, are

   a) \( \frac{1}{2}, \frac{1}{2} \)   b) \( \frac{1}{3}, \frac{2}{3} \)   c) \( \frac{1}{4}, \frac{3}{4} \)   d) \( \frac{1}{5}, \frac{4}{5} \)   e) none of the above

2. The area of the region bounded by the graphs of \( y = \sqrt{x} \) and \( y = x^2 \) is

   a) \( \frac{1}{2} \)   b) \( \frac{2}{3} \)   c) \( \frac{1}{4} \)   d) \( \frac{5}{8} \)   e) none of the above

3. \( \int_0^4 |x - 1| \, dx \) is

   a) 4   b) 2   c) 5   d) 12   e) none of the above

4. The derivative of \( F(x) = \int_x^{x^2} t^{1/2} + 1 \, dt \) is

   a) \( x^2 \sqrt{x + 1} - x \sqrt{x^{1/2} + 1} \)   b) \( (x^2 - x) \sqrt{x - x^{1/2} + 1} \)
   c) \( 2x^3 \sqrt{x + 1} - 2x^2 \sqrt{x^{1/2} + 1} \)   d) \( 2x^3 \sqrt{x + 1} - x \sqrt{x^{1/2} + 1} \)
   e) none of the above

5. \( \int x(x + 1)^{-\frac{1}{2}} \, dx \) equals

   a) \( x(x + 1)^{1/2} - \int (x + 1)^{1/2} \, dx \)   b) \( 2x(x + 1)^{1/2} - 2 \int (x + 1)^{1/2} \, dx \)
   c) \( x(x + 1)^{-1/2} - \int (x + 1)^{-1/2} \, dx \)   d) \( x^2(x + 1) \)
   e) none of the above

6. If the acceleration of a particle traveling along a line is \( a(t) = 2 + 6t \) for \( 0 \leq t \leq 10 \) and the position at \( t = 0 \) is 10 and the velocity at \( t = 0 \) is 3 then the position of the particle at \( t = 2 \) is
7. The volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{x}$, $y = 0$, and $x = 4$ about the x-axis is
   a) $8\pi$ b) $4\pi$ c) 4 d) $\pi$ e) none of the above

8. The length of the curve $y = \frac{2}{3}(x^2 + 1)^{3/2}$ for $1 \leq x \leq 4$ is
   a) 20 b) 40 c) 45 d) 10 e) none of the above

9. The average of the function $f(x) = 3x^2 + 7$ on the interval $0 \leq x \leq 2$ is
   a) 10 b) 14 c) 22 d) 11 e) none of the above

10. Let $\int_0^2 \frac{1}{\sqrt{x^2}} \, dx$ is
   a) $\infty$ b) $2^{1/3}$ c) $3(2^{1/3})$ d) $2^{-1/2}$ e) none of the above

11. $\int_1^\infty \frac{\ln x}{x} \, dx$ is
   a) 1 b) $\frac{1}{2}$ c) $\infty$ d) -1 e) none of the above

12. Which of the following integrals gives the length of the graph of $y = \sqrt{x}$ between $x = a$ and $x = b$, where $0 < a < b$?
   a) $\int_a^b \sqrt{x^2 + x} \, dx$ b) $\int_a^b \sqrt{x + \sqrt{x}} \, dx$
   c) $\int_a^b \sqrt{x + \frac{1}{2\sqrt{x}}} \, dx$ d) $\int_a^b \sqrt{1 + \frac{1}{2\sqrt{x}}} \, dx$
   e) $\int_a^b \sqrt{1 + \frac{1}{4x}} \, dx$

13. The maximum possible area of a rectangle of perimeter 200 m is

3
14. \[ \int \frac{x \, dx}{\sqrt{3x^2 + 5}} \]

a) \( \frac{1}{5}(3x^2 + 5)^{3/2} + C \)  

b) \( \frac{1}{9}(3x^2 + 5)^{3/2} + C \)  

c) \( \frac{1}{12}(3x^2 + 5)^{1/2} + C \)  

d) \( \frac{1}{4}(3x^2 + 5)^{1/2} + C \)  

e) \( \frac{2}{3}(3x^2 + 5)^{1/2} + C \)

15. The acceleration at time \( t \) of a particle moving on the x-axis is \( 4\pi \cos t \). If the velocity is 0 at \( t = 0 \), what is the average velocity of the particle over the interval \( 0 \leq t \leq \pi \)?

a) 0  

b) \( \frac{4}{\pi} \)  

c) 4  

d) 8  

e) 8\pi

16. \( \int_{0}^{\infty} e^{-2t} \, dt \) is

a) \( -\infty \)  

b) \( -\frac{1}{2} \)  

c) \( \frac{1}{2} \)  

d) 1  

e) \( \infty \)

17. If \( f \) is continuous for all \( x \), which of the following integrals necessarily have the same value?

I. \( \int_{a}^{b} f(x) \, dx \)  

II. \( \int_{0}^{b-a} f(x + a) \, dx \)  

III. \( \int_{a+c}^{b+c} f(x + c) \, dx \)

a) I and II only  

b) I and III only  

c) II and III only  

d) I, II, III  

e) No two necessarily have the same value.

18. The area of the region in the first quadrant between the graph of \( y = x\sqrt{4-x^2} \) and the x-axis is
Part B. For these problems, show your work.

1. A rectangular box with square base is constructed to have a volume of 100 cubic feet. If the material per square foot for the top of the box is twice as costly as the rest of the box, then find the dimensions of the box that minimizes the total cost of the material.

2. Evaluate the indefinite or the definite integral for each of the following

   (a) \( \int \frac{dx}{x \sqrt{\ln x}} \)

   (b) \( \int_{-\pi}^{\pi} \frac{x \cos x}{1 + x^2} \, dx \)

   (c) \( \int x \sqrt{x - 1} \, dx \)

   (d) \( \int x \sin 3x \, dx \)

3. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

   (a) \( y = x^2 + 1, \ x = 0, \ y = 5 \) about the x-axis

   (b) \( y = (x + 1)^2, \ x = 0, \ y = 0 \) about the y-axis

19. \( \int_{-2}^{1} \frac{1}{x} \, dx \) is

   a) \( -\frac{1}{2} \)
   b) \( \frac{1}{2} \)
   c) 1
   d) divergent
   e) none of the above

20. The absolute maximum of \( f(x) = x \sqrt{18 - x^2} \) on \([0, 4]\) is

   a) 0
   b) \( \sqrt{18} \)
   c) 9
   d) 5
   e) none of the above
4. Let $f$ be the function

$$f(x) = \frac{1}{2}e^{-|x|}, \quad -\infty < x < \infty$$

(a) Show that $f$ is a probability density function

(b) Find the mean value $\mu$

(c) Find $P(-1 \leq X \leq 2)$ where $X$ is the random variable with probability density $f$.

5. (a) Find an antiderivative for

$$\int \frac{1}{x^2 - 1} \, dx$$

(b) A force of 30 lb. is required to maintain a spring stretched from its natural length of 12 in. to a length of 15 in. How much work is done in stretching the spring from 12 in. to 20 in.? 