Multiple Choice Problems

1. $\int_0^r \sqrt{4-x^2} \, dx =$
   (A) $\pi$  (B) $4\pi$  (C) 7  (D) 18  (E) none of the above

2. $\int \frac{d}{dx} e^{\arctan x} \, dx =$
   (A) $\frac{1}{1+x^2} e^{\arctan x} + C$
   (B) $\frac{1}{\sqrt{1-x^2}} e^{\arctan x}$
   (C) $e^{\arctan x}$
   (D) none of the above

3. For $x^*_i$ in the interval $[0, 1]$
   \[
   \lim_{n \to \infty} \sum_{i=1}^{\infty} \left[ 2x^*_i - 3(x^*_i)^2 - \cos \left( \frac{\pi}{2} x^*_i \right) \right] \Delta x =
   \]
   (A) $-1$  (B) $\infty$  (C) 2  (D) 0  (E) none of the above

4. If $\int_0^b 6x^2 - 2bx \, dx = 64$ then $b =$
   (A) 3  (B) 8  (C) 4  (D) $2\sqrt{2}$  (E) none of the above

5. A particle moves along a horizontal line with velocity at time $t$ given by $v(t) = t^2 - 2t$. The total distance traveled by the particle from $t = 0$ to $t = 4$ is
   (A) $16/3$  (B) $-8$  (C) 8  (D) $19/3$  (E) none of the above

6. A cable weighing 20 lb/linear-ft is attached to a skid holding 1000 lbs. of ore at the bottom of a 50 ft shaft. How much work in ft-lbs is done in lifting the ore to the top of the shaft from the bottom?
   (A) $100,000$  (B) $25,000$  (C) $125,000$  (D) $75,000$  (E) none of the above

7. If $f(x)$ is an odd function and $\int_0^c f(x) \, dx = \pi$ then $\int_{-c}^c f(x) \, dx =$
   (A) $2\pi$  (B) 0  (C) $3\pi$  (D) $\pi/2$  (E) none of the above

8. If $f(x) = \sin (x^3)$ then the iterative formula for Newton’s approximation to the solution of $f(x) = 0$ is given by $x_{n+1} =$
   (A) $x_n - \frac{\sin(x_n^3)}{3x_n^2 \sqrt{1-x_n^2}}$
   (B) $x_n - \frac{3x_n^2 \sin(x_n^3)}{1+x_n^2}$
   (C) $3x_n^2 \cos(x_n^3)$
   (D) $x_n - \frac{\tan(x_n^3)}{3x_n^2}$
   (E) none of the above

9. Suppose you have exactly $360$ to spend on a rectangular container where the material for the top and bottom costs $9 /m per square cm and the material for the side costs $6 per square cm. If the length of the base must be twice the width, what is the maximum possible volume of such a container?
   (A) $\frac{20}{3}$  (B) $\frac{10}{3}$  (C) $\frac{4000}{3}$  (D) $\frac{40}{3}$
   (E) none of the above

10. Consider the function $G(x) = \int_0^x \frac{t^2}{\sqrt{x^2}} \cos t \, dt$ defined on $[0, \sqrt{\pi}]$. Where is $G$ increasing?
    (A) $(0, \frac{\pi}{2}) \cup \left( \frac{3\pi}{4}, 2\pi \right)$
    (B) $(0, \pi)$
    (C) $(0, \frac{\pi}{2}) \cup \left( \pi, \frac{3\pi}{2} \right)$
    (D) $(\frac{\pi}{4}, \frac{3\pi}{4})$
    (E) none of the above
11. Consider the region bounded by the graphs of \( h(x) = 4 - 3x \) and \( u(x) = x^2 \) where \( 0 \leq x \leq 1 \). What is the length \( L \) of the boundary of this region?

(A) \( \int_0^1 \sqrt{(x - 3)^2 + x^2} \, dx \)  
(B) \( 4 + \int_0^1 \sqrt{1 + 4x^2} \, dx \)  
(C) \( 4 + \int_0^1 \sqrt{9 + 4x^2} \, dx \)  
(D) \( 4 + \int_0^1 \sqrt{1 + 4x^2} \, dx \)  
(E) none of the above

12. The radius of a circle is increasing at the uniform rate of .5 cm/s. What is the rate of change of the circle’s area (in \( \text{cm}^2/\text{s} \)) when its radius is 117 cm?

(A) 134\( \pi \)  
(B) 234\( \pi \)  
(C) 117\( \pi \)  
(D) 117  
(E) none of the above

13. Find \( \lim_{t \to 0} \frac{e^{1/t} \sin t - 3}{t} \).

(A) \( \infty \)  
(B) \(- \infty \)  
(C) limit does not exist  
(D) 0  
(E) none of the above

14. Suppose \( f \) and \( g \) are differentiable functions with \( f(3) = 5 \), \( f'(4) = 3 \), \( g(3) = 4 \), and \( g'(3) = 8 \). Then for \( F(x) = f(g(x)) \), \( F'(3) = \)

(A) 20  
(B) 12  
(C) 24  
(D) 15  
(E) none of the above

15. Find \( \lim_{t \to 0} \frac{e^{1/t} \sin t - 3}{t} \).

(A) \( \infty \)  
(B) \(- \infty \)  
(C) limit does not exist  
(D) 0  
(E) none of the above

16. \( \int_0^2 |x^2 - 1| \, dx = \)

(A) 3  
(B) 6  
(C) 2  
(D) \( \frac{4}{3} \)  
(E) none of the above

17. A spring requires 20 N of force to compress and hold it 10 cm past rest position. How much work (in Joules= Newton-meters) is done in compressing the spring from 5 cm to 10 cm past rest position?

(A) 1 J  
(B) .01 J  
(C) 75 J  
(D) .75 J  
(E) none of the above

18. Suppose the height \( X \) (in meters) of a certain type of tree is a continuous random variable with a probability density function \( f(x) \). What is the probability that the height of a randomly chosen tree is between 10 m and 15 m?

(A) \( \int_{10}^{15} f(x) \, dx \)  
(B) \( \int_{0}^{15} f(x) \, dx \)  
(C) \( \int_{0}^{15} f(x) \, dx \)  
(D) \( \int_{0}^{15} xf(x) \, dx \)  
(E) none of the above

19. Find \( \lim_{h \to 0} \frac{e^{h} - 1}{h} \).

(A) \( \infty \)  
(B) \(- \infty \)  
(C) limit does not exist  
(D) 1  
(E) none of the above

20. If \( \int_1^4 2g(t) \, dt = 16 \) then there must be a number \( c \) such that \( g(c) = \)

(A) 4/3  
(B) 2  
(C) 4  
(D) 8  
(E) none of the above

21. If \( \int_0^1 7h(x) \, dx = \pi \) then \( \int_0^{\sqrt{7}} xh(x^2) \, dx = \)

(A) 17/2  
(B) 17  
(C) 34  
(D) 0  
(E) none of the above

22. If \( f(16) = 5 \) and \( F(x) = \int_0^x f(t) \, dt \) then \( F'(4) = \)

(A) 80  
(B) 20  
(C) 5  
(D) 40  
(E) none of the above
23. Find \( \lim_{x \to \infty} \left( \frac{1}{x} \right)^{1/x} \).

(A) 0  (B) \( \infty \)  (C) \( -\infty \)  (D) does not exist  (E) none of the above

24. Find a number \( c \) such that \( f(c) = f_{ave} \) where \( f(x) = \frac{\pi}{2} \cos x \) for \( x \) in \([0, \pi/2]\), \( c = \)

(A) \( \pi/4 \)  (B) \( \pi/2 \)  (C) \( \pi/6 \)  (D) 0  (E) none of the above

25. If \( f(x) \) is continuous for all \( x \) and \( f(1) = 4 \) then \( \lim_{x \to \sqrt{\pi-1}} \left[ f(\ln(x^2 + 1)) \right]^2 = \)

(A) 4  (B) 2  (C) 16  (D) limit does not exist  (E) none of the above

26. Suppose \( f \) is a function with a continuous derivative, \( f(-1) = 1, f(4) = 3 \) and \( \int_{-1}^{4} f(x) \, dx = 2. \) Find \( \int_{-1}^{4} xf'(x) \, dx. \)

(A) 10  (B) 13  (C) 16  (D) 11  (E) none of the above

27. If \( x^3 + y^3 = 6xy \) then \( \frac{dy}{dx} = \)

(A) \( \frac{3x^2 + 3y^2}{6y} \)  (B) \( \frac{2uy - x^2}{2y - 2x} \)  (C) \( 3x^2 + 3y^2 \)  (D) \( \frac{3x^2 + 6xy}{3y^2} \)  (E) none of the above

28. Given \( f'(t) = 1/t - t^2 \) and \( f(1) = 2 \) find \( f(t). \)

(A) \( \ln t - t^3/3 \)  (B) \( \ln t - t^3/3 + 2 \)  (C) \( \ln t - t^3/3 + 5/3 \)  (D) \( \ln t - t^3 + 3 \)  (E) none of the above

29. \( \int_{1}^{\infty} \frac{3}{x^2} \, dx \) diverges if \( p \)

(A) \( p > 1 \)  (B) \( P \geq 1 \)  (C) \( p < 1 \)  (D) \( p \leq 1 \)  (E) none of the above

Long Answer

1. At the top of a 100-ft tall building, a chain weighing 1.5-lbs/ft is attached to a tank containing 500-lbs of water. Due to a small hole, for every foot the tank rises it loses 2.5-lbs. of liquid. Find the work done in lifting the bucket to the top of the building.

2. Use the Fundamental Theorem of Calculus to find a continuous function \( f \) and a number \( a \) such that \( \int_{a}^{x} f(t) \, dt = e^{\sqrt{x}} - e^4. \)

3. Consider the region \( R \) bounded by the curves \( y = x^2, y = \sqrt{x} \) where \( x \geq 0 \)

   a. Find the area of \( R. \)

   b. Find the volume of the solid with base \( R \) whose cross-sections perpendicular to the x-axis are half-circles.

   c. Find the volume of the solid generated by revolving the region about the line \( x = -1. \)

4. Consider the continuous random variable \( X \) and the function \( f(x) = \)

\[
\begin{align*}
0 & \quad x < 0 \\
\frac{1}{2} & \quad x \geq 0
\end{align*}
\]

a. Show that \( f \) is a probability density function.

b. Find the mean value, \( \mu. \)
c. Find $P(X \leq 1)$.

5. “Gabriel’s Horn” is the solid generated by revolving the region between the line $y=0$ and the graph of $f(x) = 1/x$ on $[1, \infty)$ about the x-axis. Find the volume of this solid.

6. Suppose $H(x) = \int_0^x e^{t^3-4t} \, dt$ for all $x$.
   
   a. Find the intervals where $H$ increases and decreases.
   b. Find where $H$ is concave up and concave down.
   c. Find the local minimum and maximum values of $H$

7. Consider the integral $\int_0^4 e^{x^2} \sin (\pi x) \, dx$. With $n = 8$ setup, but do not evaluate, the following approximations to the integral:
   
   a. Midpoint
   b. Trapezoid
   c. Simpson’s

8. Use an integral to find the volume of a 10 cm tall pyramid with square base with sides 5 cm long.

9. Consider the parametric curve

   $x = 3t^2 - 1, \quad y = 4t$

   a. Find the equation of the line tangent to the curve when $t = 1$.
   b. Setup, but do not evaluate, an integral giving the length of the curve from $t = -1$ to $t = 3$.

10. Evaluate the following integrals:
    
    a. $\int_1^2 \frac{x^3-2x}{x^2} \, dx$
    b. $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{\sin x} \, dx$
    c. $\int \tan^{-1}(x) \, dx$
    d. $\int \frac{x^2+x-6}{(x+3)(x^2-1)} \, dx$
    e. $\int_0^1 \frac{1}{\sqrt{1-x}} \, dx$
    f. $\int_{\ln(3)}^{\infty} e^{-t/10} \, dt$