1. Suppose $V$ is a vector space over $\mathbb{F}$ and $W \subseteq V$ is a subspace of $V$. Prove that annihilator of the the annihilator of $W$ is itself. That is, notationally, prove that $W = W^{00}$.

2. Suppose $V$ is vector space of finite dimension, $\dim V = n$, over $\mathbb{F}$. Let $g, f_1, \ldots, f_r \in V^*$ be linear functionals. Let $N$ be the null space of $g$ and $N_i$ be the null space of $f_i$.

Then, $N_1 \cap N_2 \cap \cdots \cap N_r \subseteq N$ if and only if $g = \sum_{i=1}^{r} c_i f_i$ for some $c_i \in \mathbb{F}$.

3. Suppose $V$ is a vector space over $\mathbb{F}$ and $W \subseteq V$ is a subspace of $V$. Suppose $g_1, \ldots, g_r \in V^*$ forms a basis of the annihilator $W^0$. Write $N_i = Null(g_i)$. Prove that

$$ W = \cap_{i=1}^{r} N_i. $$

4. Suppose $V, W$ be two finite dimensional vector spaces over $\mathbb{F}$. Let $T : V \to W$ be a linear transformation and $T^t : W^* \to V^*$ be the transpose. Prove that $rank(T) = rank(T^t)$. 
Also prove that for a $m \times n$ matrix $A$ with entries in $A$, we have $row-rank(A) = column-rank(A)$.

5. Suppose $V$ is vector space of finite dimension, $\dim V = n$, over $\mathbb{F}$. Define the map

$$\varphi : L(V, V) \to L(V^*, V^*)$$

by $\varphi(T) = T^t$. Prove that $\varphi$ is an isomorphism.