Theorem 0.1 Let $V$ be a vector space over a field $F$ and $V = \text{Span}(e_1,e_2,\ldots,e_n)$. Suppose $m > n$ is an integer and $v_1,\ldots,v_m$ be $m$ elements in $V$. Then $v_1,\ldots,v_m$ are linearly dependent.

Proof. We have

$$
\begin{pmatrix}
  v_1 \\
v_2 \\
\vdots \\
v_m
\end{pmatrix}
= A
\begin{pmatrix}
  e_1 \\
e_2 \\
\vdots \\
e_n
\end{pmatrix}
$$

Where $A = (a_{ij})$ is an $m \times n$ matrix with $a_{ij} \in F$. Now the homogeneous system of equations

$$(X_1,\ldots,X_m)A = (0,0,\ldots,0)$$

has $n$ equations in $m$ unknowns. Since $m > n$, the system has a non-zero solution. That means there is a row vector $(c_1,c_2,\ldots,c_m)$, not all $c_i = 0$ and

$$(c_1,c_2,\ldots,c_m)A = (0,0,\ldots,0).$$

Therefore

$$(c_1,c_2,\ldots,c_m)
\begin{pmatrix}
v_1 \\
v_2 \\
\vdots \\
v_m
\end{pmatrix}
= (c_1,c_2,\ldots,c_m)A
\begin{pmatrix}
e_1 \\
e_2 \\
\vdots \\
e_n
\end{pmatrix}
= 0
$$

Hence $\sum_{i=1}^m c_i v_i = 0$ and $v_1,v_2,\ldots,v_m$ are linearly dependent.