Your Name: ________________________________

On this exam, you may use a calculator, but no books or notes.

It is not sufficient to just write down the answers. You must explain how you arrived at your answers and how you know they are correct.

1 (40) ________
2 (40) ________
3 (40) ________
4 (40) ________
5 (40) ________
6 (50) ________
7 (50) ________
Total (300) ________
(1) a) Show that \( F = (x^3 - 2xy^2)i - 3x^2y^2j \) is a gradient vector field.
    b) Evaluate the integral of \( F \) along the path \( x = \cos^3 \theta, \ y = \sin^3 \theta, \ 0 \leq \theta \leq \pi/2 \).

(2) Using Green’s theorem evaluate \( \int_C x^3 dy - y^3 dx \), where \( C \) is the unit circle \( x^2 + y^2 = 1 \).

(3) Evaluate the integral \( \int_S F \cdot dS \) where \( F = xi + yj + 3k \) and \( S \) is the surface of the unit sphere \( x^2 + y^2 + z^2 = 1 \).

(4) Let \( F(x, y, z) = xi + yj + zk \). Evaluate \( \int_c F \cdot ds \) where \( c(t) = (e^t, t, t^2), \ 0 \leq t \leq 1 \).

(5) Using an appropriate change of variables, evaluate the integral \( \int_B \exp[(y-x)/(y+x)]dxdy \) where \( B \) is the interior of the triangle with vertices at \( (0, 0), (0, 1), (1, 0) \).

(6) Let \( S \) be the part of the cone \( z^2 = x^2 + y^2 \) with \( z \) between 1 and 2 oriented by the normal pointing out of the cone. Compute \( \int_S F \cdot dS \) where \( F(x, y, z) = (x^2, y^2, z^2) \).

(7) Write the number 120 as a sum of three numbers so that the sum of the products taken two at a time is a maximum.