Abstract: For a finite set $S$ of positive integers, let

$$\theta(S) := \{ i \in S : i \neq 1 \text{ and } i - 1 \notin S \}.$$ 

One interpretation of $\theta$ is that it transforms the descent set of a permutation to its peak set. For instance, the descents of the permutation 321576498 occur at positions $\{1, 2, 5, 6, 8\}$ and the peaks occur at positions $\{5, 8\}$, which is $\theta(\{1, 2, 5, 6, 8\})$.

In the past few years the descents-to-peaks transform $\theta$ has shown up (sometimes in disguise) in a variety of seeming unrelated situations, notably (1) the computation of the flag vector of a zonotope (Minkowski sum of line segments) in terms of the flag vector of the associated central hyperplane arrangement; (2) the enumeration of weights of Stembridge’s enriched $P$-partitions; (3) the deformation of Solomon’s descent algebra by a $q$-Dynkin idempotent. A common thread in these topics is their connection to quasisymmetric functions, but their underlying motivations are quite different. I will discuss these known examples and give a new one involving the use of $\theta$ to compute flag vectors of certain CW spheres constructed from finite posets. I will also give an interpretation of $\theta$ as a random walk, describe its spectrum, stationary distribution, and convergence rate, and discuss the potential use of probabilistic techniques in flag vector problems.