

Problems around Corona

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The classical Carleson Corona Theorem states that if bounded analytic in the unit disc \mathbb{D} functions f_j satisfy $\sum_{j=1}^N |f_j|^2 \geq \delta^2 > 0$, then there exist bounded analytic functions $g_j \in H^\infty(\mathbb{D})$ such, that $\sum_{j=1}^N g_j f_j = 1$. This is equivalent to the fact that the unit disk \mathbb{D} is dense in the maximal ideal space of the algebra H^∞ of bounded analytic functions.

While its initial motivation was the theory of maximal ideals of H^∞ , the importance of the Corona Theorem goes much beyond it. The Corona Theorem, and especially its generalization, the so called Matrix (Operator) Corona Theorem, play an important role in operator theory (angles between invariant subspaces, unconditionally convergent spectral decompositions, computation of spectrum, etc.), control theory (stabilization by stable feedback controllers), theory of Banach algebras (visibility of spectrum).

In the talk I am going to discuss some new results related to the Corona Problem, describe connection with the operator theory and with the geometry of holomorphic vector bundles, as well as state some new (and some old) open problems.

It worth mentioning that in the area “around corona” there are much more open questions than answers. In particular, it is still an open problem to find out if the Corona Theorem holds for a polydisc or a unit ball in \mathbb{C}^n or for general domain in \mathbb{C} .

Modulation invariant bilinear operators

Andrea Nahmod
University of Massachusetts

In past few decades harmonic analysis has developed beyond the linear framework into the multilinear one where we find pioneer work by J.M. Bony and by R. Coifman and Y. Meyer. Still, questions remained about operators which could have (highly) singular multipliers or (multilinear) symbols with 'non-standard' decay conditions. In this talk we describe recent work in this direction and some open questions. We start with (translation invariant) bilinear operators with non-smooth multipliers and a comprehensive criterion in one dimensions ensuring their boundedness. The approach relies on time frequency analysis as pioneered by C. Fefferman and further developed by Lacey and Thiele. We continue with the unfolding theory of (non-translation invariant) bilinear pseudo-differential operators with x -dependent symbols; for which little is known beyond those in the Coifman-Meyer class. We will describe how the bilinear pseudo-differential setup differs from the linear one for example in terms of symbolic calculus and boundedness properties on products of Lebesgue spaces; and will explain a new boundedness criterion for bilinear pseudo-differential operators under modulation invariance.

A few remarks about the critical dissipative quasi-geostrophic equation

Fedor Nazarov
Michigan State University

We are going to discuss several questions concerning the critical dissipative quasi-geostrophic equation, especially the problem of global existence and regularity of solutions. This is a joint work with Alexander Kiselev and Alexander Volberg.

Contributed Talks

Alfonseca, Maria, North Dakota State University

Title: Geometric properties of intersection bodies

Abstract: We prove that direct sums are not intersection bodies in dimension 7 and higher. We also find a necessary condition for convex bodies of revolution with a face to be intersection bodies in terms of their smoothness. Joint work with D. Ryabogin and A. Zvavitch.

Bateman, Michael, Indiana University

Title: Kakeya Sets in Cantor Directions

Abstract: We discuss Kakeya-type sets in the plane (i.e., sets with small measure whose doubles have large(r) measure) and construct such a set out of rectangles with long side having its slope in the Cantor set. We will also mention applications to directional maximal operators and Fourier multipliers.

Bényi, Arpád, Western Washington University

Title: Time-frequency analysis of unimodular Fourier multipliers

Abstract: We show that the phase-space concentration (measured in the norm of modulation spaces) of the solutions to the free Schrödinger and wave equations are preserved. This is joint work with K. Grochenig, K. Okoudjou, and L. Rogers.

Bilyk, Dmitriy, Georgia Institute of Technology

Title: On the Small Ball Inequality in three dimensions

Abstract: We prove a non-trivial lower bound for the L^∞ norm of hyperbolic sums of Haar functions in three dimensions. This inequality substantially improves the famous result of Jozsef Beck, simplifying and extending his methods. We also discuss important connections of this inequality to questions in the Irregularities of Distribution, Approximation Theory and Probability Theory. This is joint work with Michael Lacey.

Chen, Weidong, Kansas State University

Title: The Ill-posedness of The Sampling Theorem and Regularized Sampling Algorithm

Abstract: In this paper the ill-posedness of Shannons sampling theorem is discussed and a regularized sampling algorithm for band-limited signals is presented. The convergence of the regularized sampling algorithm is studied and compared with Shannons sampling theorem by some examples.

Diestel, Geoff, University of South Carolina

Title: Maximal Bilinear Singular Integral Operators Associated with Planar Sets

Abstract: Bounds for dyadic maximal operators are obtained for dilations of polygons and certain planar sets with smooth boundaries. In the case of polygons, the dyadic nature allows for some stronger results as well.

Gatto, A. Eduardo, Institution: DePaul University

Title: Lipschitz Spaces and Singular Integrals Associated to Non-doubling Measures

Abstract: In 1916 Privalov proved that the Conjugate Function is a bounded operators on Lipschitz Spaces. In this talk we extend this result to Singular Integral Operators on a metric space with a nondoubling measure.

Goldberg, Michael, Johns Hopkins University

Title: The Schrödinger Equation with a Large Magnetic Potential

Abstract: Many estimates for solutions of the Schrödinger equation remain valid even in the presence of a large fixed magnetic field, provided the field strength decays fast enough away from the origin.

Kovalev, Leonid, Texas A&M University, Department of Mathematics

Title: Quasiregular gradient mappings

Abstract: Complex gradients of solutions of certain elliptic PDE are quasiregular mappings. However, not every quasiregular mapping is a gradient, and I am interested in the properties specific to gradients. Partly based on joint work with Al Baernstein II.

Oh, Choonghong (Hiro), Dept of Math & Stats, UMASS, Amherst

Title: On the local well-posedness of a one parameter family of coupled KdV systems

Abstract: We consider a system of two coupled nonlinear equations of Korteweg-de Vries (KdV) type arising in oceanography and introduced by A. Majda and J. Biello. Of special interest in its applications is the periodic set-

up. The well-posedness theory in the case of a single KdV equation is fully developed. This theory, however, does not extend to the system under consideration. The highest derivatives in the system are coupled by a parameter α . If a certain number C related to the coupling constant $0 < \alpha < 1$ is rational we show that particular resonances occur and that the crucial estimates needed to prove well-posedness fail completely. If the constant C is irrational, then these estimates hold but the results vary, depending sensitively on ‘how irrational’ C is. While this kind of phenomena occurs in mechanical systems, especially for the persistence of periodic solutions (KAM theory), it is somewhat peculiar for the well-posedness theory of partial differential equations to be so sensitive to the rational and irrational characters of a coupling constant.

Pereyra, Maria Cristina, University of New Mexico

Title: Haar multipliers meet Bellman functions

Abstract: We obtain the optimal rate of decay for the operator bound in L^2 of Haar multipliers associated to a weight with respect to the reverse Hölder characteristic of the weight. Techniques used involve Bellman functions.

Peterson, James, Benedictine College

Title: Geometric Measure Theory and Scans

Abstract: I discuss a new type of object in geometric measure theory, scans, a generalization of currents based on the notion of looking at the slices of a current. Some compactness results and uses will be stated.

Reguera Rodriguez, Maria Carmen, University of Missouri

Title: Is the characteristic function of the paraboloid a bounded bilinear multiplier?

Abstract: We consider bilinear Fourier multiplier operators on $\mathbb{R}^2 \times \mathbb{R}^2$ whose symbol is the characteristic function of the paraboloid $P = \{(\xi, \eta) \in \mathbb{R}^2 \times \mathbb{R}^2 : \xi_2 > \xi_1^2 + \eta_1^2 + \eta_2^2\}$. We use a Keakeya type counterexample to show that such a bilinear operator is unbounded from $L^p(\mathbb{R}^2) \times L^q(\mathbb{R}^2)$ to $L^r(\mathbb{R}^2)$ outside the local L^2 case, i.e. the case when one of p , q , or $r' = r/(r-1)$ is less than 2.

Urbina, Wilfredo, University of New Mexico

Title: Fractional Integration and Fractional Differentiation for Jacobi expansions

Abstract: In this work we define and study the Fractional Integral, the Frac-

tional Derivative and the Bessel potentials for the Jacobi operator. We also obtain a characterization of the potential spaces and a version of Calderons reproduction formula.

Uriarte-Tuero, Ignacio, University of Missouri Columbia

Title: Improved Painlevé removability for quasiregular mappings

Abstract: I will present the results of a joint work with Astala, Clop, Mateu and Orobitg, in trying to understand the removable sets for bounded quasiregular mappings, and some sharp examples.

Wang, Lianwen, Central Missouri State University

Title: Existence and uniqueness for a class of BVP Problems

Abstract: The existence and uniqueness of solutions for a class of boundary value problems is discussed. Those BVP problems in the abstract form of partial differential equations are generalizations of the so-called Hamiltonian systems which are important for many applications such as optimal control, mathematical finance, differential games, and so on. Several existence and uniqueness results of the BVP problems are established by means of homotopy method under some natural assumptions.

Wick, Brett, Vanderbilt University

Title: Multi-Parameter Riesz Commutators

Abstract: It is shown that product BMO can be characterized by the multi-parameter commutators of Riesz transforms. This is joint work with M. Lacey, J. Pipher and S. Petermichl.