A report on health care in the US said that 28% of Americans have experienced times when they haven't been able to afford medical care. A news organization randomly sampled 801 black Americans, of whom 38% reported that there had been times in the last year when they had not been able to afford medical care. Does this indicate that this problem is more severe among black Americans?

1. Test an appropriate hypothesis and state your conclusion. (Make sure to check any necessary conditions and to state a conclusion in the context of the problem.)

\[ H_0: \; \pi = 0.28 \quad \text{Data:} \; n = 801 \quad \text{We expect} \; n\pi = (801)(0.28) = 224.28 \text{ successes} \]

\[ H_1: \; \pi > 0.28 \quad \text{We observe} \; \hat{p} = 0.38 \quad \text{and} \; n(1-\pi) = (801)(0.72) = 576.72 \text{ failures} \]

Test statistic: \[ Z_{obs} = \frac{\hat{p} - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{0.38 - 0.28}{\sqrt{\frac{(0.28)(0.72)}{801}}} = 6.30 \]

\[ P\text{-value} = P(Z \geq 6.30) = 1.5 \times 10^{-10} \quad --- \text{essentially zero}. \]

Conclusion: The P-value is very, very low. I reject \( H_0 \). There is strong evidence that the problem of being unable to afford health care is more severe among black Americans than among white Americans.

A one-tail upper tail test at a 95% confidence level has a critical region of \( [Z_{0.05}, \infty) \).

\[ Z_{0.05} = 1.645 \quad \text{Zobs} = 6.30 > Z_{0.05} = 1.645, \text{ so the conclusion is to reject } H_0. \]

2. Was your test one-tail upper tail, one-tail lower tail, or two-tail? Explain why you chose that kind of test in this situation.

I used an upper tail test because the observed proportion was far in excess of the expected proportion. I used a P-value test to gain an idea of the likelihood of obtaining \( Z_{obs} = 6.30 \).

3. Explain what your P-value means in this context.

In this context, the P-value is the probability of observing a Z-value at or above \( Z_{obs} = 6.3 \), given that the true proportion, \( \pi \), is equal to 0.28%.

The observed P-value of \( 1.5 \times 10^{-10} \) means that there is essentially no way the observed proportion came from a distribution with \( \pi = 0.28 \).
In 2000, the United Nations claimed that there was a higher rate of illiteracy in men than in women from the country of Qatar. A humanitarian organization went to Qatar to conduct a random sample. The results revealed that 45 out of 234 men and 42 out of 251 women were classified as illiterate on the same measurement test. Do these results indicate that the United Nations findings were correct?

1. Test an appropriate hypothesis and state your conclusion.

Let $P_m$ be the illiteracy rate in men, and let $P_w$ be the illiteracy rate in women.

$H_0: P_m = P_w$

$H_1: P_m > P_w$

Test statistic: $Z_{obs} = \frac{(\hat{P}_m - \hat{P}_w) - (P_m - P_w)}{\sqrt{\frac{P_m(1-P_m)}{n_m} + \frac{P_w(1-P_w)}{n_w}}}$

Assuming $P_m = P_w = p$ and $q_m = q_w = q$

$Z_{obs} = (\hat{P}_m - \hat{P}_w) / \sqrt{pq(1/n_m + 1/n_w)}$

Estimate $p = \frac{X_m + X_w}{n_m + n_w} = \frac{45 + 42}{234 + 251} = 0.179$

$q = (1-p) = 1 - 0.179 = 0.821$

$Z_{obs} = \frac{0.192 - 0.167}{\sqrt{(0.179)(0.821)(1/234 + 1/251)}} = 0.718$

One sided upper tail test at 95% CL has $Z_{0.05} = 1.645$

$Z_{obs} = 0.718 \neq Z_{0.05} = 1.645$ so $H_0$ should be retained.

$p$-value = $P(Z > Z_{obs}) = 0.236$ The $p$-value is large so $H_0$ should be retained.

Conclusion: The upper tail 95% confidence level test and the upper tail $p$-value test are in agreement. There is insufficient evidence to conclude that there is a higher illiteracy rate in men than in women.

2. Find a 95% confidence interval for the difference in the proportions of illiteracy in men and women from Qatar. Interpret your interval.

$95\%$ CL so $\alpha = 0.05$ $Z_{0.05} = Z_{0.025} = 1.96$

$(\hat{P}_m - \hat{P}_w) \pm Z_{0.05} \sqrt{\frac{\hat{P}_m(1-P_m)}{n_m} + \frac{\hat{P}_w(1-P_w)}{n_w}} = 0.025 \pm 0.068$

95% confidence interval for $P_m - P_w$: $[-0.043, 0.093]$

The 95% confidence interval contains the value 0, which suggests that there is essentially no difference between men and women in terms of the illiteracy rate.
Textbook authors must be careful that the reading level of their book is appropriate for the target audience. Some methods of assessing reading level require estimating the average word length. We've randomly chosen 20 words from a randomly selected page in *Stats: Data and Models* and counted the number of letters in each word:

5, 5, 2, 11, 1, 5, 3, 8, 5, 4, 7, 2, 9, 4, 8, 10, 4, 5, 6, 6

The sampled words have a mean length of 5.5 letters and a standard deviation of 2.685 letters.

1. Suppose that our editor was hoping that the book would have a mean word length of 6.5 letters. Does this sample indicate that the authors failed to meet this goal? Test an appropriate hypothesis and state your conclusion.

\[ H_0 : \mu = 6.5 \]
\[ H_1 : \mu \neq 6.5 \]

A plot of the data shows it to be reasonably close to the normal distribution, \( \sigma \) is unknown.

Test statistic:

\[ t_{obs} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = -1.6656 \]

Decision criterion: Reject \( H_0 \) if \( |t_{obs}| > t_{0.025} \) (\( \nu = 19 \) degrees of freedom).

\[ t_{0.025} (\nu = 19) = 2.093 \]

Since \( |t_{obs}| > t_{0.025} \), there is insufficient evidence to conclude that the mean word length is not 6.5 letters. The test was conducted at a 95% confidence level.

2. For a more definitive evaluation of reading level the editor wants to estimate the text's mean word length to within 0.5 letters with 98% confidence. How many randomly selected words does she need to use?

\[ E > Z_{0.01} \frac{S}{\sqrt{n}} \Rightarrow n > \frac{Z_{0.01}^2 S^2}{E^2} \]

\[ n > \frac{(2.326)^2 (2.685)^2}{0.5^2} = 156 \]

She needs 156 or more words.
A professor at a large university believes that students take an average of 15 credit hours per term. A random sample of 24 students in her class of 250 students reported the following number of credit hours that they were taking:

\[
\begin{array}{cccccccccccc}
12 & 13 & 14 & 14 & 15 & 15 & 16 & 16 & 16 & 16 & 16 & 16 \\
17 & 17 & 17 & 18 & 18 & 18 & 19 & 19 & 19 & 20 & 21 \\
\end{array}
\]

The sample has a mean of 16.6 credit hours and a standard deviation of 2.22 credit hours.

1. Does this sample indicate that students are taking more credit hours than the professor believes? Test an appropriate hypothesis and state your conclusion.

\[
\begin{align*}
H_0 & : \mu = 15 \\
H_1 & : \mu > 15 \\
\text{Data: } & \bar{x} = 16.6, \quad n = 24 \\
\text{Test statistic: } & t_{\text{obs}} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = 3.5308 \\
\end{align*}
\]

Upper tail test at 95% confidence level. \( t_{0.05}(r = 23) = 1.714 \)

Conclusion:

\( t_{\text{obs}} > t_{0.05} \), so, at a 95% confidence level, there is sufficient evidence to conclude that the students are taking more credit hours than the professor hypothesized.

2. Find a 95% confidence interval for the number of credit hours taken by the students in the professor's class. Interpret your interval.

\[
\bar{x} \pm t_{(n-1), 0.025} \frac{s}{\sqrt{n}} = 16.6 \pm (2.069) \frac{2.22}{\sqrt{24}} = 16.6 \pm 0.938 \text{ credit hours}
\]

It appears that the students in the professor's class are taking more than 15 credit hours on average.
A random sample of 13 men and 19 women in a college class reported their grade point averages (GPAs). Here are histograms from the data:

Summary statistics for these data are:

<table>
<thead>
<tr>
<th></th>
<th>( \bar{Y} )</th>
<th>( s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>2.898</td>
<td>0.583</td>
</tr>
<tr>
<td>Women</td>
<td>3.330</td>
<td>0.395</td>
</tr>
</tbody>
</table>

\( n_m = 13 \)
\( n_w = 19 \)

1. A woman in the class says that she believes that college women tend to have higher GPAs than do college men. Does this sample support her claim? Test an appropriate hypothesis and state your conclusion.

\[
\begin{align*}
H_0 & : \mu_m = \mu_w \\
H_1 & : \mu_w > \mu_m
\end{align*}
\]

Test statistic

\[
t_{\text{obs}} = \frac{\left( \bar{Y}_w - \bar{Y}_m \right) - (\mu_w - \mu_m)}{\sqrt{\frac{s_w^2}{n_w} + \frac{s_m^2}{n_m}}} \\
(\text{with degrees of freedom} \ \nu = n_w - 1) \quad \nu = \frac{(s_w^2/n_w)^2 + (s_m^2/n_m)^2}{(s_w^2/n_w + s_m^2/n_m)^2} \quad \nu \approx n_w - 1
\]

Upper tail test. 95% CL.

\[t_{0.05}(\nu=19) = 1.729 \quad t_{\text{obs}} = 2.3306\]

Conclusion: \( t_{\text{obs}} > t_{0.05}(\nu=19) \) so there is sufficient evidence to support the claim that college women average a higher GPA than do college men.

2. Create and interpret a 95% confidence interval.

\[
\left( \bar{Y}_w - \bar{Y}_m \right) \pm t_{0.025}(\nu=19) \sqrt{\frac{s_w^2}{n_w} + \frac{s_m^2}{n_m}} = 0.432 \pm 0.38795
\]

We are 95% confident that the number of excess grade points that women have over men lies in the interval \([0.04405, 0.81995]\). There appears to be a small, but real difference between the men’s and women’s GPAs.
Past experience indicates that the time required for high school seniors to complete a standardized test is a normal random variable with a standard deviation of 6 minutes. Test the hypothesis that $\sigma = 6$ against the alternative that $\sigma < 6$ if a random sample of 20 high school seniors has a standard deviation $s = 4.51$. Use a 0.05 level of significance.

Null hypothesis: $H_0: \sigma = 6$

Alternative hypothesis: $H_1: \sigma < 6$

Test statistic: Chi-Squared $\chi^2 = \frac{(n-1)s^2}{\sigma^2} = 10.735$ with 19 degrees of freedom

$\chi^2(\nu = 19) = 10.117$

$\chi^2_{obs} < \chi^2_{0.05}$ so there is insignificant evidence to refute the null hypothesis that $\sigma = 6$ minutes at a 95% confidence level. However, even though $\chi^2_{obs}$ did not fall within the critical region, it came quite close. More data should be obtained and the hypothesis that $\sigma < 6$ minutes should be retested.

References:

Our class textbook: Probability & Statistics for Engineers & Scientists by Walpole, Myers, Myers, & Ye.

Our class notes: lectures by Prof. Pasik-Duncan

Computations performed with GNU Octave, FreeMat, & WxMaxima.

This is my own work.

Signature: Jeremy Ims