Midterm 1 Conceptual Review
Math 146: Honors Calculus II, Spring 2016

All material covered in class, from the text or supplemental, is fair game for the midterm. For a comprehensive discussion of the topics covered, see the Daily Updates.

The sections covered from the text are §§1.7, 5.5 - 5.7, 5.10, and 6.1 - 6.7. However, we went much more in-depth, learning more sophisticated methods and solving harder problems, than the text covers on many of these topics.

Remember that showing your steps is necessary to earn credit for solving problems on the exam.

The following list is a general guide to the basic topics covered, but is not an exhaustive list of the types of problems that can appear. Applying the concepts to solve new, challenging types of problems is a major goal of an honors mathematics course.

Note that the solutions to new problems are not given; please come to office hours, discuss with classmates, or post on Piazza if you would like to verify your methods.

Levels of understanding. Ask the following questions to identify your mastery of each concept:

1. Can I complete a problem from an example in the text correctly, including the same amount of exposition as in the text, without looking at the text?
2. Can I solve early problems from the sections in the text quickly?
3. Can I do all the homework problems, including the most difficult ones?
4. Can I figure out some of the additional challenge problems below?

Integration techniques. (§§5.5 - 5.7, and more advanced versions of methods in §5.7)

- **Methods**: Substitution, integration by parts, integrals involving (powers of) trigonometric functions, trigonometric substitution, partial fractions decomposition
- **Goals**: Identify which technique(s) should be used in a specific setting, apply the techniques for improper and definite integrals, use a procedure more than once if necessary, identify appropriate bounds when doing a trigonometric substitution
- **Additional problems**: Chapter 5 Review: #25, 29, 31, 33, 34, 36
  - Find the antiderivatives of the following functions of $x$:

    \[
    \tan^2 x, \sqrt{x} \ln(x), \sqrt{x^2 - 1}, \frac{1}{x \sqrt{x^2 + 1}}, \frac{2x + 1}{x^3 - 3x^2 + 3x - 1}, \frac{x}{\sqrt{1 - x^2}}, \sec^3 x, \\
    \cos(\ln(x)), \frac{2x^2 + x + 1}{(x + 3)(x - 1)^2}, \ln(\cos(x)) \tan(x), x^3 e^{x^2}, e^{ax} \sin(bx), \frac{1}{x \sqrt{x^2 - 1}}, \frac{\sqrt{1 - x}}{1 - \sqrt{x}}, \\
    \frac{x + 4}{x^2 + 1}, \frac{2x}{(x^2 + x + 1)^3}, \arctan(x), \frac{1}{1 + x^2}, \arcsin(\sqrt{x}), \frac{8x^2 + 6x + 4}{x + 1},
    \]
Improper integrals. (§5.10)

- **Goals**: Identify whether an integral is improper, identify why an improper integral is improper, turn an improper integral into a limit, decide whether an improper integral converges or diverges by evaluating a limit or by using the Comparison Theorem, evaluate convergent improper integrals, apply L'Hôpital's rules to evaluate limits
- **Additional problems**: §5.10: #49, 59, 62, 64, 66; Chapter 5 Review: #56, 57, 59, 62
  - Decide whether the following integrals converge or diverge:
    \[
    \int_{-\infty}^{\infty} \frac{1}{|x|} \, dx, \quad \int_{1}^{\infty} \frac{e^x}{e^{3x} + 2} \, dx, \quad \int_{1}^{\infty} \frac{e^x}{e^{3x} - 2} \, dx, \quad \int_{0}^{\infty} \frac{e^x}{e^x + x^2} \, dx, \quad \int_{0}^{\infty} \frac{\sin \left( \frac{1}{x} \right)}{\sqrt[5]{x}} \, dx
    \]

Areas and average value. (§§6.1, 6.5)

- **Goals**: Find the area between curves, find the average value of a function, use the Mean Value Theorem for Integrals, find the specified value in the Mean Value Theorem
- **Additional problems**: §6.1: #37, 39, 40, 42, 44; §6.5: #8, 10, 22

Volumes. (§§6.2 - 6.3)

- **Methods**: Disk method, washer method, cylindrical shell method
- **Goals**: Find the volume of a solid obtained by rotating an area about a horizontal or vertical line using the three basic methods, or using basic geometry; find the volume of a geometric solid using integration
- **Additional problems**: §6.2: #18, 26, 32, 34, 37, 40, 49; §6.3: #18, 24, 35, 36, 38
  - Find the volume and surface area of the “infinite trumpet” obtained by revolving the graph of \( f(x) = \frac{1}{x} \) for \( x \geq 1 \) about the \( x \)-axis.

Arc length. (§6.4)

- **Goals**: Write an integral expression for the length of a (bounded) curve
- **Additional problems**: §6.4: #27, 30, 32

Parametric curves. (§§1.7, 6.1, 6.4 and additional material on tangent lines)

- **Goals**: Sketch parametric curves, identify formulas from graphs of parametric curves, find the points where the tangent line to a parametric curve achieves a certain value, find the area inside a parametric curve or between two parametric curves, find the arc length of a bounded parametric curve
- **Additional problems**: §1.7: #21, 22, 34; §6.1: #34, 35; §6.4: #10, 16, 22
Applications to physics and engineering. (§6.6, additional material on centers of mass)

- Concepts: Work done, hydrostatic force and pressure, moments and centers of mass
- Goals: Model a problem related to the concepts, use calculus to solve the problem
- Additional problems: §6.6: Find the center of mass for the regions in #47 and #48 with density constant, and then $\rho(x) = x^2$; #52

Applications to economics and biology. (§6.7)

- Concepts: Consumer surplus, blood flow, cardiac output
- Goals: Model a problem related to the concepts, use calculus to solve the problem
- Additional problems: #3, 6, 14