

# EXAM 3 REVIEW - MATH 101

The following is a review of the concepts you should know for this exam. This review is a sample and is not intended to mirror the exam questions. Any problems similar to those discussed in class or in assigned homework may also be assessed on the exam.

- Given  $f(x) = x^2 + 6x - 1$ ,
  - Identify the vertex of the graph of  $f(x)$ .
  - What is the axis of symmetry?
  - Identify the  $y$ -intercept of the graph of  $f(x)$ .
  - Does the function have a maximum or a minimum?
  - Make a careful labeled sketch of the graph of  $f(x)$ .
- Find the quadratic function with vertex at  $(-1, 4)$  and passing through  $(5, -5)$ .
- The profit that a vendor makes per day by selling  $x$  pretzels is given by the function  $P(x) = -0.002x^2 + 1.2x - 150$ . Find the number of pretzels that must be sold to maximize profit.
- A developer wants to enclose a rectangular grassy lot that borders a city street for parking. If the developer has 284 feet of fencing and does not fence the side along the street, what is the largest area that can be enclosed?
- The quadratic function  $f(x) = 0.041x^2 - 0.45x + 30.06$  models the median age at which men in the United States were first married  $x$  years after 1990. In which year was this average age at a minimum? (Round to the nearest year.) What was the average age at first marriage for that year? (Round to the nearest tenth.)
- For the polynomial, describe the end behavior of the graph. List each real zero and its multiplicity. Determine whether the graph crosses or touches the  $x$ -axis at each  $x$ -intercept.
  - $f(x) = 5(x + 1)^2(x - 5)(x + 2)^3$
  - $g(x) = 4(x^2 + 1)(x - 2)^3$
  - $h(x) = 2x^3 - 6x$
- Find the domain of the following functions:
  - $f(x) = \frac{x + 7}{x^2 - 25}$
  - $g(x) = \frac{x - 8}{x^2 - 4x}$
  - $h(x) = \frac{-3x^2}{x^2 + 3x - 88}$
- Find the horizontal or oblique asymptotes for the function.
  - $f(x) = \frac{x^2 + 8x - 6}{x - 6}$
  - $g(x) = \frac{x(x - 1)}{2x^2 - 3x}$
- Find the  $x$ - and  $y$ -intercepts of the graph of the following rational functions.
  - $f(x) = \frac{x - 8}{3x - 14}$
  - $g(x) = \frac{(5x - 20)(x - 6)}{x^2 + 24x - 16}$
  - $h(x) = \frac{23}{(x + 10)(x^2 - 2)}$
  - $j(x) = \frac{x^2 - 8x}{x^2 - 10x - 4}$

10. Make a careful sketch of the graph of the given rational function. Label all intercepts, asymptotes and at least 3 additional points.
- a)  $f(x) = \frac{2x}{(x-3)(x-1)}$
- b)  $g(x) = \frac{x}{x^2-36}$
11. Determine if the graph of  $f(x) = \frac{x^2-7x+10}{(x-4)^2}$  intersects the horizontal or oblique asymptote. [Hint: Expand the denominator first.]
12. The illumination provided by a car's headlight varies inversely as the square of the distance from the headlight. A car's headlight produces an illumination of 5 footcandles at a distance of 44 feet. What is the illumination when the distance is 55 feet?
13. Use long division or synthetic division to find the quotient when the polynomial  $P(x) = 8x^4 + 23x^3 - 3x^2 + x + 4$  is divided by  $x + 3$ . Write your answer in the form (quotient)(divisor) + remainder.
14. Is  $(x + 1)$  a factor of  $-3x^4 - 2x + 1$ ? Explain.
15. Is  $\frac{1}{2}$  a zero of  $4x^3 - 5x^2 - 3x + 1$ ? Explain.
16. List all the potential rational zeros for the polynomial:  $f(x) = 6x^4 + 4x^3 - 2x^2 + 2$ .
17. Use the given zero to find all the zeros of the polynomial. Then write  $P(x)$  in factored form over the reals.
- a)  $P(x) = x^4 - 6x^3 + 7x^2 - 6x + 6$  with zero  $i$
- b)  $P(x) = 2x^4 + 2x^3 - 11x^2 + x - 6$  with zero 2
- c)  $P(x) = x^3 - 4x^2 + 4x - 16$  with zero  $2i$
18. Find a polynomial with integer coefficients that has the following zeros: .5, 3, 1 (with multiplicity 2).
19. Find all zeroes of  $P(x) = 4x^4 - 2x^3 - 28x^2 + 5x + 42$  and write  $P(x)$  in factored form.
20. In interval notation, the domain of  $\frac{1}{x-2}$
- a)  $(-\infty, 2] \cup [2, \infty)$     b)  $(-\infty, 2) \cup (2, \infty)$     c)  $(-\infty, \infty)$     d)  $(2, \infty)$
21. The vertex of the parabola,  $f(x) = 2(x-3)^2 - 5$  is:
- a) (3, 5)    b) (-3, 5)    c) (-3, -5)    d) (3, -5)
22. In using synthetic division, the divisor is always a polynomial of degree 1, whose leading coefficient is 1.
- a) True    b) False
23. Given a polynomial  $P(x) = 2x^2 - 4x - 15$ . Finding  $P(2)$  by substitution gives the same result as dividing  $P(x)$  by  $(x-2)$
- a) True    b) False

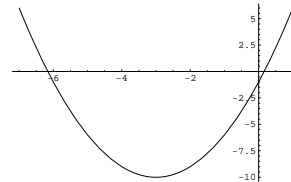
24. The maximum or minimum of a parabola always occurs at the  $x$ -axis.

- a) True                                      b) False

25. If  $y = \frac{3}{x}$ , it can be said that

- a)  $y$  varies inversely as  $x$                                       b)  $y$  varies directly as  $x$   
 c)  $y$  varies jointly to  $x$     d)  $y$  varies reversely proportional to  $x$

**Answers:**



1. a) (-3, -10)      b)  $x = -3$       c) -1      d) Minimum      e)

2.  $f(x) = -\frac{1}{4}(x + 1)^2 + 4$

3. 300 pretzels

4. 10,082 square feet

5. 1995; 28.8 years old

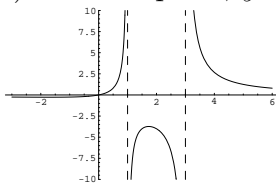
6. a) -1, mult 2 (touch); 5, mult 1 (cross); -2, mult 3 (cross)  
 b) 2, mult 3 (cross)  
 c) 0, mult 1 (cross);  $-\sqrt{3}$ , mult 1 (cross);  $\sqrt{3}$ , mult 1 (cross)

7. a)  $\{x|x \neq \pm 5\}$  or  $(-\infty, -5) \cup (-5, 5) \cup (5, \infty)$   
 b)  $\{x|x \neq 0, 4\}$  or  $(-\infty, 0) \cup (0, 4) \cup (4, \infty)$   
 c)  $\{x|x \neq 8, -11\}$  or  $(-\infty, -11) \cup (-11, 8) \cup (8, \infty)$

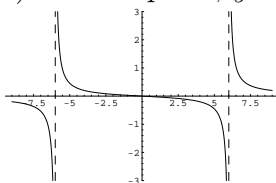
8. a) OA:  $y = x + 14$       b) HA:  $y = \frac{1}{2}$

9. a)  $x$ -intercept: 8;  $y$ -intercept:  $\frac{4}{7}$                                       b)  $x$ -intercepts: 4, 6;  $y$ -intercept:  $-\frac{15}{2}$   
 c) No  $x$ -intercept;  $y$ -intercept:  $-\frac{23}{20}$                                       d)  $x$ -intercepts: 0, 8;  $y$ -intercept: 0

10. a)  $x$ -intercept: 0;  $y$ -intercept: 0; Asymptotes:  $x = 3, x = 1, y = 0$



b)  $x$ -intercept: 0;  $y$ -intercept: 0; Asymptotes:  $x = 6, x = -6, y = 0$



11. Yes, the graph intersects the horizontal asymptote at  $(6, 1)$ .
12. 3.20 footcandles
13.  $(8x^3 - x^2 + 1)(x + 3) + 1$
14. Yes. Using the Factor Theorem, division by  $(x + 1)$  results in a remainder of 0
15. No - Using the Remainder Theorem, substituting  $\frac{1}{2}$  for  $x$  does not result in a value of 0.
16.  $\pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm \frac{2}{3}$
17. Zeros: a)  $i, -i, 3 \pm \sqrt{3}$     b)  $2, -3, -\frac{\sqrt{2}}{2}i, \frac{\sqrt{2}}{2}i$     c)  $2i, -2i, 4$
18.  $P(x) = 2x^4 - 11x^3 + 19x^2 - 13x + 3$
19. Zeros:  $-2, \frac{3}{2}, \frac{1 \pm \sqrt{15}}{2}$ ;  $P(x) = (4)(x + 2)(x - \frac{3}{2})(x - \frac{1+\sqrt{15}}{2})(x - \frac{1-\sqrt{15}}{2})$ ; Note: There are other ways to write the factored form, but you should account for the leading coefficient.
20.  $b$
21.  $d$
22.  $a$
23.  $a$
24.  $b$
25.  $a$