

# EXAM 4 REVIEW - MATH 101

The following is a review of the concepts you should know for this exam. This review is a sample and is not intended to mirror the exam questions. In addition, any problems similar to those discussed in class or in assigned homework may be assessed on the exam.

- Let  $f(x) = 3x - 5$  and  $g(x) = -2x + 7$  and find the following:
  - $(g \circ f)(x)$
  - $(g \circ g)(x)$
  - $(f \circ g)(-2)$
- Given  $H(x) = \frac{3}{\sqrt{2x+5}}$ , find non-trivial functions  $f(x)$  and  $g(x)$  so that  $(f \circ g)(x) = H(x)$ .

In the following problems, each of the functions is one-to-one. Find the inverse, graph  $f$  and  $f^{-1}$  on the same coordinate system, and state the domain and range for both functions.

- $f(x) = -1 - 3x$
- $f(x) = x^2 + 3, x \geq 0$
- $f(x) = x^3 - 1$

Use transformations to graph the exponential and logarithmic functions in the following problems. Identify the domain, range, and asymptotes for each. Label the intercepts and other key points on the graphs.

- $f(x) = 3^{x+2}$
- $g(x) = -e^x + 1$
- $h(x) = \log_5(x - 1)$
- $k(x) = \ln x - 2$

Expand the given expressions into a series of sums and differences of logarithms. Express powers as factors.

- $\log \sqrt{ab^2c^3}$
- $\ln \frac{(x+2)}{(x-5)^2}$
- $\log_3 \frac{3}{\sqrt{xy}}$

Write the following expressions as single logarithms.

- $2 \log_3 x + 4 \log_3 y - 3 \log_3 z$
- $\ln m + 2 \ln n - 3 \ln k - \frac{1}{2} \ln j$

Solve for  $x$  in the following equations. Give both an exact answer and an approximation correct to 3 decimal places

- $9^{3x} = 27^{x-5}$
- $\left(\frac{1}{2}\right)^{x-2} = 32^{4-x}$
- $5^{x+1} = 4^{2x-3}$
- $e^{2x} = 7$
- $\ln 5x = 4$
- $\log_7(2x+1) = 3$
- $2 \log x + \log 5 = \log 15 + \log 2x$
- $\log_5(x-3) + \log_5(x-4) = \log_5 8$
- $3^{2x-5} = 5^{-x+4}$

24. Suppose you borrow \$15,000 at 6.75% interest compounded continuously. How much will you owe in 4 years? Assuming you make no payments, how many years (to the nearest year) will it take for the amount due to be \$22,500?
25. Sue just inherited \$3,500 which she decides to invest in an account that pays 3% compounded quarterly. How much will be in the account in 3 years? How long will it take to double the investment?
26. Jack and Diane are planning to buy a home and they know they will need at least \$15,000 for a downpayment. The best investment they can find now is paying 5.5% compounded continuously. How much should they invest now in order to have the necessary amount for the downpayment in 5 years?
27. There are 15 angel fish in a fish tank at the beginning of the month. Thirty days later there are 55 angel fish. How many days will it be before there are 100 angel fish?
28. A leg bone is found on an archeological dig and is found to contain only 15% of its normal amount of carbon-14. Considering the half-life of carbon-14 to be 5730 years, how old is the bone (to the nearest year?)
29. The half-life of the radioactive isotope carbon-14 is approximately 5730 years. Carbon-14 is present in all living organisms. When the organism dies, the element decays. If a living tissue shows a radioactivity count of 15.3 dmp per gram of carbon-14 and a sample of the same tissue only contains 5.2 dpm per gram of carbon-14, how old is the object?
30. A metal object is heated to 110° F and then allowed to cool in a room with temperature 60° F. If the object is 100° F after 10 minutes, when will the temperature of the object measure 80° F?
31. If the inverse of a given function is also a function, then :
- |                               |                               |
|-------------------------------|-------------------------------|
| a) the function is a parabola | b) the function is a cubic    |
| c) the function is a line     | d) the function is one-to-one |
32. If the point (1, 3) is one the graph of  $f$ , what point must be on  $f^{-1}$ ?
- |           |             |
|-----------|-------------|
| a) (1, 3) | b) (-1, -3) |
| c) (3, 1) | d) (-3, -1) |
33. In the exponential growth model  $A(t) = A_0e^{kt}$ , the value "k" represents:
- |                     |                   |
|---------------------|-------------------|
| a) initial amount   | b) final amount   |
| c) length of growth | d) rate of growth |

**Answers:**

1. a)  $-6x + 17$     b)  $4x - 7$     c) 28

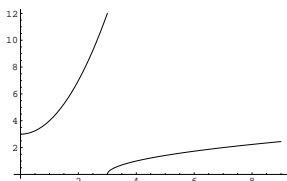
2.  $f(x) = \frac{3}{x}, g(x) = \sqrt{2x + 5}$  is one example

3. Inverse:  $f^{-1}(x) = -\frac{1}{3}x - \frac{1}{3}$ ; Domain of  $f$ :  $(-\infty, \infty)$ ; Range of  $f$ :  $(-\infty, \infty)$ ;  
 Domain of  $f^{-1}$ :  $(-\infty, \infty)$ ; Range of  $f^{-1}$ :  $(-\infty, \infty)$   
 check on your graphing calculator

4. Inverse:  $f^{-1}(x) = \sqrt{x - 3}$ ;

Domain of  $f$ :  $[0, \infty)$ ; Range of  $f$ :  $[3, \infty)$ ;

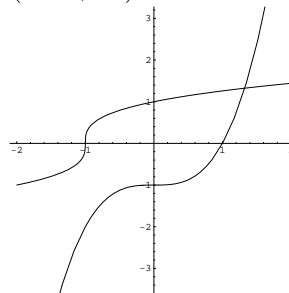
Domain of  $f^{-1}$ :  $[3, \infty)$ ; Range of  $f^{-1}$ :  $[0, \infty)$



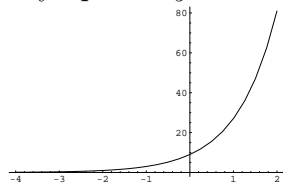
5. Inverse:  $f^{-1}(x) = \sqrt[3]{x + 1}$ ;

Domain of  $f$ :  $(-\infty, \infty)$ ; Range of  $f$ :  $(-\infty, \infty)$ ;

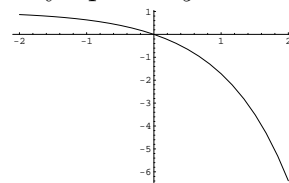
Domain of  $f^{-1}$ :  $(-\infty, \infty)$ ; Range of  $f^{-1}$ :  $(-\infty, \infty)$



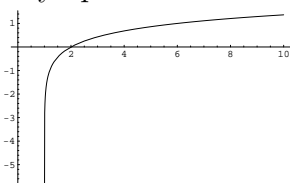
6. Domain:  $(-\infty, \infty)$ ; Range:  $(0, \infty)$ ;  
 Asymptote:  $y = 0$



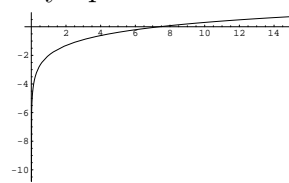
7. Domain:  $(-\infty, \infty)$ ; Range:  $(-\infty, 1)$ ;  
 Asymptote:  $y = 1$



8. Domain:  $(1, \infty)$ ; Range:  $(-\infty, \infty)$ ;  
 Asymptote:  $x = 1$



9. Domain:  $(0, \infty)$ ; Range:  $(-\infty, \infty)$ ;  
 Asymptote:  $x = 0$



10.  $\frac{1}{2}(\log a + 2 \log b + 3 \log c)$

11.  $\ln(x+2) - 2\ln(x-5)$       12.  $1 - \frac{1}{2}(\log_3 x + \log_3 y)$       13.  $\log_3 \frac{x^2 y^4}{z^3}$

14.  $\ln \frac{mn^2}{k^3 \sqrt{j}}$       15.  $x = -5$       16.  $x = 4.5$

17.  $x = \frac{\ln 5 + 3 \ln 4}{2 \ln 4 - \ln 5} \approx 4.96$       18.  $x = \frac{1}{2} \ln 7$       19.  $x = \frac{1}{5} e^4$

20.  $x = 171$       21.  $x = 6$       22.  $x = \frac{7 + \sqrt{33}}{2}$

23. exact:  $x = \frac{5 \ln(3) + 4 \ln(5)}{2 \ln(3) + \ln(5)}$ ; approx:  $x = 3.134$

24. \$19,649.47; 6 years

25. \$3828.32; 23 years

26. \$11,393.58

27. 44 days

28. 15682 years

29. 8921.3 years ago or about 6913 BC

30. After 41.1 minutes

31. d

32. c

33. d