1. (a) \( \lim_{x \to -2} f(x) = 3 \)
   \( \lim_{x \to -3} f(x) \) does not exist since the left and right limits are not equal. (The left limit is \(-2\).)
   \( \lim_{x \to 0} f(x) = 4 \)
   (b) The equations of the horizontal asymptotes are \( y = -1 \) and \( y = 4 \).
   (c) The equations of the vertical asymptotes are \( x = 0 \) and \( x = 2 \).
   (d) \( f \) is discontinuous at \( x = -3, 0, 2, \) and \( 4 \). The discontinuities are jump, infinite, infinite, and removable, respectively.

2. \( \lim_{x \to -\infty} f(x) = -2 \), \( \lim_{x \to 0} f(x) = 0 \), \( \lim_{x \to 3} f(x) = \infty \),
   \( \lim_{x \to -3} f(x) = -\infty \), \( \lim_{x \to 3} f(x) = 2 \),
   \( f \) is continuous from the right at 3.

3. Since the exponential function is continuous, \( \lim_{x \to 1} e^{3-x} = e^{1-1} = e^0 = 1 \).

4. Since rational functions are continuous, \( \lim_{x \to 3} \frac{x^2 - 9}{x^2 + 2x - 3} = \lim_{x \to 3} \frac{3^2 - 9}{3^2 + 2(3) - 3} = \frac{0}{12} = 0 \).

5. \( \lim_{x \to 3} \frac{x^2 - 9}{x^2 + 2x - 3} = \lim_{x \to 3} \frac{(x+3)(x-3)}{(x+3)(x-1)} = \lim_{x \to 3} \frac{x-3}{x-1} = \frac{-3-3}{-3-1} = \frac{-6}{-4} = \frac{3}{2} \).

6. \( \lim_{x \to 1^-} \frac{x^2 - 9}{x^2 + 2x - 3} = -\infty \) since \( x^2 + 2x - 3 \to 0 \) as \( x \to 1^- \) and \( \frac{x^2 - 9}{x^2 + 2x - 3} < 0 \) for \( 1 < x < 3 \).

7. \( \lim_{h \to 0} \frac{(h-1)^3 + 1}{h} = \lim_{h \to 0} \frac{h^3 - 3h^2 + 3h - 1 + 1}{h} = \lim_{h \to 0} \frac{h^3 - 3h^2 + 3h}{h} = \lim_{h \to 0} (h^2 - 3h + 3) = 3 \)

   Another solution: Factor the numerator as a sum of two cubes and then simplify.
   \( \lim_{h \to 0} \frac{(h-1)^3 + 1}{h} = \lim_{h \to 0} \frac{(h-1)^3 + 1^3}{h} = \lim_{h \to 0} \frac{[(h-1) + 1] [(h-1)^2 - 1(h-1) + 1^2]}{h}
   = \lim_{h \to 0} \frac{[(h-1)^2 - h + 2]}{h} = 1 - 0 + 2 = 3 \)
8. \( \lim_{t \to -2} \frac{t^2 - 4}{t^3 - 8} = \lim_{t \to -2} \frac{(t + 2)(t - 2)}{(t - 2)(t^2 + 2t + 4)} = \lim_{t \to -2} \frac{t + 2}{t^2 + 2t + 4} = \frac{2 + 2}{4 + 4 + 4} = \frac{4}{12} = \frac{1}{3} \)

9. \( \lim_{r \to 9} \frac{\sqrt{r}}{(r - 9)^4} = \infty \) since \((r - 9)^4 \to 0\) as \(r \to 9\) and \(\frac{\sqrt{r}}{(r - 9)^4} > 0\) for \(r \neq 9\).

10. \( \lim_{v \to 4^+} \frac{4 - v}{|4 - v|} = \lim_{v \to 4^+} \frac{4 - v}{(4 - v)} = \lim_{v \to 4^+} \frac{1}{1} = -1 \)

11. \( \lim_{u \to 1} \frac{u^4 - 1}{u^3 + 5u^2 - 6u} = \lim_{u \to 1} \frac{(u^2 + 1)(u^2 - 1)}{u(u^2 + 5u - 6)} = \lim_{u \to 1} \frac{(u^2 + 1)(u + 1)(u - 1)}{u(u + 6)(u - 1)} = \lim_{u \to 1} \frac{(u^2 + 1)(u + 1)}{u(u + 6)} = \frac{2(2)}{1(7)} = \frac{4}{7} \)

12. \( \lim_{x \to 3} \frac{\sqrt{x + 6} - x}{x^3 - 3x^2} = \lim_{x \to 3} \left[ \frac{\sqrt{x + 6} - x}{x(x - 3)} \cdot \frac{x(x + 3)}{\sqrt{x + 6} + x} \right] = \lim_{x \to 3} \frac{(x + 6)^{1/2} - x}{x^2(x - 3)(\sqrt{x + 6} + x)} \]
\(= \lim_{x \to 3} \frac{x + 6 - x^2}{x^2(x - 3)(\sqrt{x + 6} + x)} = \lim_{x \to 3} \frac{-x^2 - x + 6}{x^2(x - 3)(\sqrt{x + 6} + x)} = \lim_{x \to 3} \frac{-x(x - 3)(x + 2)}{x^2(x - 3)(\sqrt{x + 6} + x)} \]
\(= \lim_{x \to 3} \frac{-x + 2}{x^2(\sqrt{x + 6} + x)} = -\frac{5}{9(3 + 3)} = -\frac{5}{54} \)

13. Let \(t = \sin x\). Then as \(x \to \pi^-, \sin x \to 0^+\), so \(t \to 0^+\). Thus, \(\lim_{x \to \pi^-} \ln(\sin x) = \lim_{t \to 0^+} \ln t = -\infty \).

14. \(\lim_{x \to -\infty} \frac{1 - 2x^2 - x^4}{5 + x - 3x^3} = \lim_{x \to -\infty} \frac{(1 - 2x^2 - x^4)/x^4}{(5 + x - 3x^3)/x^3} = \lim_{x \to -\infty} \frac{1/x^4 - 2/x^2 - 1}{5/x^3 + 1/x^3 - 3} = \frac{0 - 0 - 1}{0 + 0 - 3} = -1 = \frac{1}{3} \)

15. Since \(x\) is positive, \(\sqrt{x^2} = |x| = x\). Thus,
\(\lim_{x \to \infty} \frac{\sqrt{x^2 - 9}}{2x - 6} = \lim_{x \to \infty} \frac{\sqrt{x^2 - 9}/x^2}{(2x - 6)/x^2} = \lim_{x \to \infty} \frac{\sqrt{1 - 9/x^2}}{2 - 6/x} = \frac{\sqrt{1 - 0}}{2 - 0} = \frac{1}{2} \)

16. Let \(t = x - x^2 = x(1 - x)\). Then as \(x \to \infty\), \(t \to -\infty\), and \(\lim_{x \to \infty} e^{x-x^2} = \lim_{t \to -\infty} e^t = 0\).

17. \(\lim_{x \to \infty} \left[ (\sqrt{x^2 + 4x + 1} - x) \right] = \lim_{x \to \infty} \left[ \frac{\sqrt{x^2 + 4x + 1} - x}{1} \cdot \frac{\sqrt{x^2 + 4x + 1} + x}{\sqrt{x^2 + 4x + 1} + x} \right] = \lim_{x \to \infty} \frac{x^2 + 4x + 1 - x^2}{\sqrt{x^2 + 4x + 1} + x} \]
\(= \lim_{x \to \infty} \frac{(4x + 1/x)}{\sqrt{x^2 + 4x + 1} + x} \quad \text{[divide by } x = \sqrt{x^2} \text{ for } x > 0] \)
\(= \lim_{x \to \infty} \frac{4 + 1/x}{\sqrt{1 + 4/x + 1/x^2} + 1/x} = \frac{4 + 0}{\sqrt{1 + 0 + 0} + 1} = \frac{4}{2} = 2 \)

18. \(\lim_{x \to 1} \left[ \frac{1}{x - 1} + \frac{1}{x^2 - 3x + 2} \right] = \lim_{x \to 1} \left[ \frac{1}{x - 1} + \frac{1}{(x - 1)(x - 2)} \right] = \lim_{x \to 1} \left[ \frac{x - 2}{(x - 1)(x - 2)} + \frac{1}{(x - 1)(x - 2)} \right] \]
\(= \lim_{x \to 1} \frac{x - 1}{(x - 1)(x - 2)} = \lim_{x \to 1} \frac{1}{x - 2} = \frac{1}{1 - 2} = -1 \)
21. Since \( 2x - 1 \leq f(x) \leq x^2 \) for \( 0 < x < 3 \) and \( \lim_{x \to 1} (2x - 1) = 1 = \lim_{x \to 1} x^2 \), we have \( \lim_{x \to 1} f(x) = 1 \) by the Squeeze Theorem.

22. Let \( f(x) = -x^2 \), \( g(x) = x^2 \cos\left(\frac{1}{x^2}\right) \) and \( h(x) = x^2 \). Then since \( \left|\cos\left(\frac{1}{x^2}\right)\right| \leq 1 \) for \( x \neq 0 \), we have
\[
f(x) \leq g(x) \leq h(x)
\]
for \( x \neq 0 \), and so \( \lim_{x \to 0} f(x) = \lim_{x \to 0} h(x) = 0 \) \( \Rightarrow \lim_{x \to 0} g(x) = 0 \) by the Squeeze Theorem.

23. (a) \( f(x) = \sqrt{-x} \) if \( x < 0 \), \( f(x) = 3 - x \) if \( 0 \leq x < 3 \), \( f(x) = (x - 3)^2 \) if \( x > 3 \).

(i) \( \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (3 - x) = 3 \)

(ii) \( \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \sqrt{-x} = 0 \)

(iii) Because of (i) and (ii), \( \lim_{x \to 0} f(x) \) does not exist.

(iv) \( \lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (3 - x) = 0 \)

(v) \( \lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x - 3)^2 = 0 \)

(vi) Because of (iv) and (v), \( \lim_{x \to 3} f(x) = 0 \).

(b) \( f \) is discontinuous at 0 since \( \lim_{x \to 0} f(x) \) does not exist.

(c) \( f \) is discontinuous at 3 since \( f(3) \) does not exist.

24. (a) \( x^2 - 9 \) is continuous on \( \mathbb{R} \) since it is a polynomial and \( \sqrt{x} \) is continuous on \([0, \infty)\), so the composition \( \sqrt{x^2 - 9} \) is continuous on \( \{ x \mid x^2 - 9 \geq 0 \} = (-\infty, -3] \cup [3, \infty) \). Note that \( x^2 - 2 \neq 0 \) on this set and so the quotient function
\[
g(x) = \frac{\sqrt{x^2 - 9}}{x^2 - 2}
\]
is continuous on its domain, \((-\infty, -3] \cup [3, \infty)\).

(b) \( \sin x \) is continuous on \( \mathbb{R} \) by Theorem 7 in Section 2.5. Since \( e^x \) is continuous on \( \mathbb{R} \), \( e^{\sin x} \) is continuous on \( \mathbb{R} \) by Theorem 9 in Section 2.5. Lastly, \( x \) is continuous on \( \mathbb{R} \) since it's a polynomial and the product \( xe^{\sin x} \) is continuous on its domain \( \mathbb{R} \) by Theorem 4 in Section 2.5.

25. \( f(x) = 2x^3 + x^2 + 2 \) is a polynomial, so it is continuous on \([-2, -1]\) and \( f(-2) = -10 < 0 < 1 = f(-1) \). So by the Intermediate Value Theorem there is a number \( c \) in \((-2, -1)\) such that \( f(c) = 0 \), that is, the equation \( 2x^3 + x^2 + 2 = 0 \) has a root in \((-2, -1)\).

26. \( f(x) = e^{-x^2} - x \) is continuous on \( \mathbb{R} \) so it is continuous on \([0, 1]\). \( f(0) = 1 > 0 > 1/e - 1 = f(1) \). So by the Intermediate Value Theorem, there is a number \( c \) in \((0, 1)\) such that \( f(c) = 0 \). Thus, \( e^{-c^2} - c = 0 \), or \( e^{-c^2} = c \), has a root in \((0, 1)\).
33. (a) $f'(r)$ is the rate at which the total cost changes with respect to the interest rate. Its units are dollars/(percent per year).

(b) The total cost of paying off the loan is increasing by $1200/(percent per year)$ as the interest rate reaches 10%. So if the interest rate goes up from 10% to 11%, the cost goes up approximately $1200.

(c) As $r$ increases, $C$ increases. So $f'(r)$ will always be positive.
39. \( f \) is not differentiable: at \( x = -4 \) because \( f \) is not continuous, at \( x = -1 \) because \( f \) has a corner, at \( x = 2 \) because \( f \) is not continuous, and at \( x = 5 \) because \( f \) has a vertical tangent.
42. Let $C(t)$ be the function that denotes the cost of living in terms of time $t$. $C'(t)$ is an increasing function, so $C''(t) > 0$. Since the cost of living is rising at a slower rate, the slopes of the tangent lines are positive but decreasing as $t$ increases. Hence, $C'''(t) < 0$.

43. (a) $f'(x) > 0$ on $(-2, 0)$ and $(2, \infty)$ $\Rightarrow f$ is increasing on those intervals. $f'(x) < 0$ on $(-\infty, -2)$ and $(0, 2)$ $\Rightarrow f$ is decreasing on those intervals.

(b) $f'(x) = 0$ at $x = -2, 0$, and 2, so these are where local maxima or minima will occur. At $x = \pm 2$, $f'$ changes from negative to positive, so $f$ has local minima at those values. At $x = 0$, $f'$ changes from positive to negative, so $f$ has a local maximum there.

(c) $f'$ is increasing on $(-\infty, -1)$ and $(1, \infty)$ $\Rightarrow f'' > 0$ and $f$ is concave upward on those intervals.

(d) $f'$ is decreasing on $(-1, 1)$ $\Rightarrow f''' < 0$ and $f$ is concave downward on this interval.

44. (a) \hspace{10cm} (b) possible graph of $f$

45. $f(0) = 0, f'(-2) = f'(1) = f'(9) = 0, \lim_{x \to -\infty} f(x) = 0, \lim_{x \to 6} f(x) = -\infty$,

$f'(x) < 0$ on $(-\infty, -2), (1, 6)$, and $(9, \infty), f'(x) > 0$ on $(-2, 1)$ and $(6, 9), f''(x) > 0$ on $(-\infty, 0)$ and $(12, \infty), f'''(x) < 0$ on $(0, 6)$ and $(6, 12)$