Math 126: Quiz 5

Name: Anary Kolasinski
Score: 4/10 + 1 = 5/10

Show all of you work on all of the problems to get full credit.
WRITE THE TEST YOU USE TO PROVE CONVERGENCE OR DIVERGENCE.

1) Determine whether the series converge or diverge. If it converges, find its sum.

$$(4) \sum_{n=1}^{\infty} \frac{4^{n+1}}{9^n}$$

**Be careful of the starting point of $n$ ($n = 1$).

$$\frac{4^{n+1}}{9^n} = 4 \cdot \left(\frac{4}{9}\right)^n$$ so (4) is a geometric series with $a = 4$, $r = \frac{4}{9}$

Since $\frac{4}{9} < 1$ we know it converges. It converges to:

$$\sum_{n=1}^{\infty} 4 \left(\frac{4}{9}\right)^n = -4 + \sum_{n=0}^{\infty} 4 \left(\frac{4}{9}\right)^n = -4 + \frac{4}{1 - \frac{4}{9}} = \frac{16}{5}$$

2) Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{3^n + n}{8^n + n}$$

$$\lim_{n \to \infty} a_n = \lim_{n \to \infty} \left(\frac{3^n + n}{8^n + n}\right)$$

$$= \lim_{n \to \infty} \left(\frac{\left(\frac{3}{8}\right)^n + \frac{n}{8^n}}{1 + \frac{n}{8^n}}\right)$$

$$= \lim_{n \to \infty} \left(\frac{\left(\frac{3}{8}\right)^n}{\left(\frac{8}{3}\right)^n}\right)$$

$$= \lim_{n \to \infty} \left(\frac{1}{\left(\frac{8}{3}\right)^n}\right)$$

$$= 0$$

so $a_n$ converges to $0$. 
3) Consider the following series.

\[ \sum_{k=6}^{\infty} \frac{k^3 - k}{3k^3 + 7k^2 - 12} \]

Determine whether the series is convergent or divergent. If it is convergent, find its sum.

Let \( a_k = \frac{k^3 - k}{3k^3 + 7k^2 - 12} \)

\[
\lim_{k \to \infty} a_k = \lim_{k \to \infty} \frac{k^3 - k}{3k^3 + 7k^2 - 12} = \frac{1}{3} \neq 0
\]

So, by the test for divergence, \( \sum_{k=6}^{\infty} a_k \) diverges.

4) Find the values of \( x \) for which the series converges.

\[ \sum_{n=0}^{\infty} \frac{(x+2)^n}{5^n} \]

\[
\frac{(x+2)^n}{5^n} = \left( \frac{x+2}{5} \right)^n
\]

so the series is a geometric series which converges if \( \left| \frac{x+2}{5} \right| = |r| < 1 \) so

\[-1 < \frac{x+2}{5} < 1 \]

\[-5 < x+2 < 5 \]

\[-7 < x < 3 \]

so the series converges on \((-7, 3)\)

Extra Question: Who did Kansas play in their Elite 8 game last Saturday? Who should have won the game?

Villanova (played)

"KU should have won."