Bivariate Data and Linear Regression

1. A random sample of ordered pairs of points is taken from a population.

<table>
<thead>
<tr>
<th>x</th>
<th>1.0</th>
<th>2.3</th>
<th>3.1</th>
<th>4.8</th>
<th>5.6</th>
<th>6.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>2.6</td>
<td>2.8</td>
<td>3.1</td>
<td>4.7</td>
<td>5.1</td>
<td>5.3</td>
</tr>
</tbody>
</table>

(a) Find the “best fit” regression line \( y = ax + b \) and the correlation coefficient \( r \).

(b) Predict the \( y \)-value when \( x = 7.2 \).

(c) Find: \( \bar{x}, S_{xx}, \bar{y}, S_{yy}, S_{xy}, \text{SSE}, S^2 \), and \( \text{df} \).

Denote the true regression line as \( y = Ax + B \).

(d) Find a 96% confidence interval for \( A \).

(e) Find the P-value for the test that \( A > .4 \).

(f) Find a 94% confidence interval for \( B \).

Let \( x^* = 5 \).

(g) Find a 98% confidence interval for all of the \( y \)-values in the population that corresponds to \( x^* = 5 \).

(h) Find a 92% prediction interval for the particular \( y \)-value corresponding to \( x^* = 5 \).

Answers

(a) \( y = ax + b = .584x + 1.684 \), \( r = .974 \), (b) \( y(7.2) = 5.89 \)

(c) \( \bar{x} = 3.85 \), \( S_{xx} = 21.055 \), \( \bar{y} = 3.933 \), \( S_{yy} = 7.573 \), \( S_{xy} = 12.3 \), \( \text{SSE} = .388 \), \( S^2 = .097 \), and \( \text{df} = 4 \).

(d) \( a \pm t_{\frac{\alpha}{2}} \frac{S}{\sqrt{S_{xx}}} \), (.381, .788) is a 96% confidence interval for \( A \).

(e) \( T_{\text{test}} = \frac{a - A_0}{S/\sqrt{S_{xx}}} = 2.7109 \), P-value = P(T>2.7109)=.0267.

(f) \( b \pm t_{\frac{\alpha}{2}} S \sqrt{\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}} \), (9281, 2.4399) is 94% confidence interval for \( B \).

(g) \( ax^* + b \pm t_{\frac{\alpha}{2}} S \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{S_{xx}}} \), (4.045, 5.163) is a 98% confidence interval for all \( y \)-values in the population that correspond to \( x^* = 5 \).

(h) \( ax^* + b \pm t_{\frac{\alpha}{2}} S \sqrt{\frac{n+1}{n} + \frac{(x^* - \bar{x})^2}{S_{xx}}} \), (3.798, 5.4710) is a 92% prediction interval for the particular \( y \)-value that corresponds to \( x^* = 5 \).