Math 365 - Handout for Chapter 9

Start with a normal population \( n(\mu, \sigma) \) and a small sample \( X_1, X_2, \cdots, X_n \). Compute \( \bar{x} \) and \( s \).

(I) Estimating \( \mu \). We use the t-distribution; this probability density function (the random variable is denoted as \( T \)) depends on \( n - 1 \) (the degrees of freedom of the distribution and denoted as \( df = n - 1 \)). The graph of the t-distribution is a symmetrical bell shape curve, and as in the case of the normal distribution, for \( \alpha < .5 \), \( t_\alpha \) is defined as the point on the graph such that the area to the right of \( t_\alpha \) is \( \alpha \), i.e., \( P(T > t_\alpha) = \alpha \).

Computing \( t_\alpha \): Use TI-83 or TI-84. \( t_\alpha = \text{solve(tcdf}(X, 100, df) - \alpha, X, \{100, 100\}) \)

Confidence Interval: A 100\((1 - \alpha)\)% confidence interval for \( \mu \) is \( (\bar{x} - t_\frac{\alpha}{2} \frac{s}{\sqrt{n}}, \bar{x} + t_\frac{\alpha}{2} \frac{s}{\sqrt{n}}) \).

Hypothesis Testing: Compute \( T_{test} = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} \).

\[ H_0 : \mu = \mu_0 \text{ vs } H_1 : \mu < \mu_0 \quad \text{P-value} = P(T < T_{test}). \]

\[ H_0 : \mu = \mu_0 \text{ vs } H_1 : \mu > \mu_0 \quad \text{P-value} = P(T > T_{test}). \]

\[ H_0 : \mu = \mu_0 \text{ vs } H_1 : \mu \neq \mu_0 \quad \text{P-value} = 2P(T > |T_{test}|). \]

Reject \( H_0 \) at \( \alpha \) level of significance for \( \alpha \geq \text{P-value} \).

(II) Estimating \( \sigma \). We use the \( \chi^2 \)-distribution; this probability density function (the random variable is denoted as \( \chi^2 \)) depends on \( n - 1 \) (the degrees of freedom of the distribution and denoted as \( df = n - 1 \)). The graph of the \( \chi^2 \)-distribution is a curve to the right of \( x = 0 \). For \( \alpha < .5 \), \( \chi^2_\alpha \) is defined as the point on the graph such that the area to the right of \( \chi^2_\alpha \) is \( \alpha \), i.e., \( P(\chi^2 > \chi^2_\alpha) = \alpha \), and \( \chi^2_{1-\alpha} \) is defined as the point on the graph such that the area to the right of \( \chi^2_{1-\alpha} \) is \( 1 - \alpha \), i.e., \( P(\chi^2 > \chi^2_{1-\alpha}) = 1 - \alpha \).

Computing \( \chi^2_\alpha \) and \( \chi^2_{1-\alpha} \): Use TI-83 or TI-84: \( \chi^2_\alpha = \text{solve}(\chi^2\text{cdf}(X, 100, df) - \alpha, X, \{-100, 100\}) \) and \( \chi^2_{1-\alpha} = \text{solve}(\chi^2\text{cdf}(-100, X, df) - \alpha, X, \{-100, 100\}) \).

Confidence Interval: A 100\((1 - \alpha)\)% confidence interval for \( \sigma \) is \( \left( s\sqrt{\frac{n-1}{\chi^2_{\frac{\alpha}{2}}}}, s\sqrt{\frac{n-1}{\chi^2_{1-\frac{\alpha}{2}}}} \right) \).

Hypothesis Testing: Compute \( \chi^2_{test} = \frac{s^2(n-1)}{\sigma_0^2} \).

\[ H_0 : \sigma = \sigma_0 \text{ vs } H_1 : \sigma < \sigma_0 \quad \text{P-value} = P(\chi^2 < \chi^2_{test}) \]

\[ H_0 : \sigma = \sigma_0 \text{ vs } H_1 : \sigma > \sigma_0 \quad \text{P-value} = P(\chi^2 > \chi^2_{test}). \]

\[ H_0 : \sigma = \sigma_0 \text{ vs } H_1 : \sigma \neq \sigma_0 \quad \text{P-value} = 2\min\{1-P(\chi^2 < \chi^2_{test}), P(\chi^2 < \chi^2_{test})\}. \]

Reject \( H_0 \) at \( \alpha \) level of significance for \( \alpha \geq \text{P-value} \).