Read Sections 3.3 through 3.5 of the textbook (Hankerson, et al.). While you’re reading, do all the exercises. Most of them are quick and easy, and most of them have answers in the back of the book. When they ask you to repeat the same exercise on many different codes, quit when you feel you have mastered it. You are responsible for knowing how to do all the things in the exercises. Write up and hand in solutions to the following exercises.

You must show all your work and justify all your answers.

1. Use the Gilbert-Varshamov inequality to show that there exists a binary linear \((15, 6, 5)\) code.

2. (a) The Gilbert-Varshamov inequality guarantees the existence of a binary linear \((8, 3, 4)\) code. Give a parity check matrix for such a code.

   (b) Prove there is no binary linear \((8, 3, 5)\) code. Warning: the fact that these parameters do not satisfy the Gilbert-Varshamov inequality does not prove it.

3. Prove that the maximum dimension \(k\) of a binary linear code with length \(n = 15\) and distance \(d = 3\) is \(k = 11\).

4. The theorems of Section 3.1 are not enough to tell us exactly the maximum dimension of a binary linear code with length \(n = 12\) and distance \(d = 4\). Let \(K\) be this maximum dimension. Give the best lower and upper bounds for \(K\) based on the theorems of Section 3.1.

5. Sec. 3.1, p. 70, #3.1.22 (a,b,c)

6. Sec. 3.2, p. 72, #3.2.6

Extra credit:

Sec. 3.1, p. 70: #3.1.22 (d)

Hint. Let \(|C| = 2^k\) for the largest value of \(k\) that satisfies the hypothesis of the Gilbert-Varshamov inequality.

Additional extra credit problem:
Prove: If \(n - k + \binom{n-k}{3} \geq n\), then there exists a binary linear \((n, k, 4)\) code.