Read Sections 3.5 through 3.7 of the textbook (Hankerson, et al.). While you’re reading, do all the exercises. Most of them are quick and easy, and most of them have answers in the back of the book. When they ask you to repeat the same exercise on many different codes, quit when you feel you have mastered it. You are responsible for knowing how to do all the things in the exercises. Write up and hand in solutions to the following exercises.

Exercises in the textbook

Sec. 3.3, p. 74: #3.3.4, 3.3.7
For #3.3.4, use the parity check matrix $H'$ given in Exercise #3.3.6 rather than $H$ in example #3.3.1.

Sec. 3.4, p. 76: #3.4.3, 3.4.4, 3.4.5

Sec. 3.5, p. 79: #3.5.1, 3.5.2

In addition:

1. Prove: The maximum dimension of a binary linear code of length $2^r - 1$ and distance 3 is $2^r - 1 - r$.

2. Prove: If Ham$_r$ is a Hamming code of length $2^r - 1$, then the vector of all ones (of length $2^r - 1$) is in Ham$_r$.

3. Prove: If Ham$_r$ is a Hamming code, and $v$ is a codeword, then $\bar{v}$ is a codeword, where $\bar{v}$ is obtained from $v$ by changing all 0s to 1s and all 1s to 0s. (You can use #1, even if you didn’t succeed in proving #1.)

Extra credit:

1. Show that the number of codewords of weight 3 in the Hamming code of length $2^r - 1$ is $(2^r - 1)(2^r - 2)/6$.

2. Let $C_r = \text{Ham}_r$, and consider the dual code $C_r^\perp$.

   (a) Show that $C_3^\perp$ has distance 4 and that, in fact, all nonzero codewords of $C_3^\perp$ are of weight 4.

   (b) Show that $C_r^\perp$ has distance $2^{r-1}$ and that, in fact, all nonzero codewords of $C_r^\perp$ are of weight $2^{r-1}$.