Read Sections 4.5 and 5.1 of the textbook (Hankerson, et al.). While you’re reading, do all the exercises. Most of them are quick and easy, and most of them have answers in the back of the book. When they ask you to repeat the same exercise on many different codes, quit when you feel you have mastered it. You are responsible for knowing how to do all the things in the exercises. Write up and hand in solutions to the following exercises.

You must show all your work and justify all your answers.

Exercises in the textbook

Sec. 4.3, pp. 103–104: #4.3.5 (b), 4.3.6 (a), 4.3.8
Sec. 4.4, p. 106: #4.4.7, 4.4.8

In addition:

1. Prove: If $C$ is a linear cyclic code, and $C$ detects the error pattern $e$, then $C$ detects $\pi(e)$.

2. Find generator polynomials for three linear cyclic codes of length 15, one each of dimension 5, 6, and 9.

3. Is there a linear cyclic code of length 11 and dimension 5? If there is, give a generating polynomial for one. If not, explain why not.

4. Find the dimensions of all cyclic linear codes of length
   (a) 17  (b) 27  (c) 10

5. Let $C$ be the linear cyclic code of length 7 with generator polynomial $g(x) = 1 + x + x^2 + x^4$.
   (a) Give the check polynomial for $C$.
   (b) Use the check polynomial to determine if the following represent codewords in $C$: $w_1(x) = 1 + x^2 + x^5 + x^6$ and $w_2(x) = 1 + x^4 + x^5$.

Extra credit:

1. Prove: If $C$ is a linear cyclic code, and $C$ corrects the error pattern $e$, then $C$ corrects $\pi(e)$.

2. Prove: Let $C$ be a linear cyclic code of length $n \geq 3$ with generator polynomial $g(x)$ of degree at least 1. If $n$ is the least positive integer such that $g(x)$ divides $1 + x^n$, then $C$ has minimum distance at least 3.