Math 601 Homework Assignment #11 Due Wednesday, April 20, 2016

Read Sections 5.1 through 5.3 of the textbook (Hankerson, et al.). While you’re reading, do all the exercises. Most of them are quick and easy, and most of them have answers in the back of the book. When they ask you to repeat the same exercise on many different codes, quit when you feel you have mastered it. You are responsible for knowing how to do all the things in the exercises. Write up and hand in solutions to the following exercises.

You must show all your work and justify all your answers.

Exercises in the textbook

Sec. 4.5, p. 110 #4.5.5 (f,h)
Sec. 5.1, pp. 112–113: #5.1.5, 5.1.10 (a,b)

In addition:

1. For each of the following \((n, k)\), answer the following, justifying your answers.

   (a) Is there a linear cyclic code of length \(n\) and dimension \(k\)?
   (b) If there is such a code, give the generator polynomial and check polynomial for one such code \(C\).
   (c) For \(C\) in (b), give a parity-check matrix for \(C\).
   (d) for \(C\) in (b), give a generator polynomial for \(C^\perp\).

   (i) \((n, k) = (15, 9)\)  \(\) (ii) \((n, k) = (13, 8)\)  \(\) (iii) \((n, k) = (14, 9)\)

2. Let \(C\) be the linear code with generator matrix

\[
G = \begin{bmatrix}
1 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 1 \\
\end{bmatrix}.
\]

   (a) Show \(C\) is cyclic.
   (b) Show \(C\) is a Hamming code.

3. (a) Prove: Let \(C\) be a linear cyclic code and suppose \(C\) contains a codeword of odd weight. Then the generator polynomial of \(C\) corresponds to a codeword of odd weight.

   (b) Prove: For each \(n \geq 3\), there is a polynomial (in \(K^n[x]\))

\[
f(x) = f_0 + f_1 x + \cdots + f_k x^k \text{ of degree } k \geq 1 \text{ such that } f(x) \text{ divides } 1 + x^n \text{ and } (f_0 f_1 \cdots f_k) \text{ has odd weight.}
\]

Extra credit problems on next page
Extra credit:
  Sec. 4.4, p. 105: #4.4.10

Additional extra credit problem.
  Prove: There is no (13, 6, 5) binary linear code.
  Note. This is not about cyclic codes.
  Hint. Suppose such a code exists. Then there is an equivalent code with a
  generator matrix whose first row is 1111100000000. Show that this implies there is
  an (8, 5, 3) binary linear code, and find a contradiction.