

## Review of Chapter 4: Applications of the Derivative

**Keywords:** applications

### 1. Formulas

### 2. Concepts, theorems, and techniques

#### 2.1 Graph of a function.

- Critical points: find points where  $f'(x) = 0$  or  $f'(x)$  is discontinuous or not defined.
- Intervals where  $f(x)$  is increasing/decreasing: intervals where  $f'(x) > 0/f'(x) < 0$ . Key is to find the critical points.
- Concavity of  $f(x)$ .  
Concave upward:  $f'(x)$  is increasing or  $f''(x) > 0$ .  
Concave downward:  $f'(x)$  is decreasing or  $f''(x) < 0$ .
- Intervals where  $f(x)$  is concave upward/downward: key is to find the points where  $f''(x) = 0$  or  $f''(x)$  is discontinuous or not defined.
- Inflection points: points where the concavity of  $f(x)$  changes.

#### 2.2 Optimization.

- Find the relative (or local) extrema.
  1. First derivative test. Find the critical points and check the sign change of  $f'(x)$  around these points.  
Let  $x = c$  is a critical point.
    - If  $f'(x)$  is increasing around  $x = c$ ,  $f(c)$  is a relative minimum.
    - If  $f'(x)$  is decreasing around  $x = c$ ,  $f(c)$  is a relative maximum.
  2. The second derivative test.  $x = c$  is a critical point and  $f'(c) = 0$ .
    - If  $f''(c) > 0$ , then  $f(c)$  is a relative minimum.
    - If  $f''(c) < 0$ , then  $f(c)$  is a relative maximum.
    - If  $f''(c) = 0$ , cannot be determined.
- Find the absolute (or global) extrema. Compares the values of  $f(x)$  at the critical points, the end-points of the domain, and possibly  $\lim_{x \rightarrow -\infty} f(x)$  and/or  $\lim_{x \rightarrow +\infty} f(x)$ .
- Optimization application problems.
  1. Set up mathematical model (or formulas).
  2. Eliminate one variable.
  3. Find the absolute maximum and/or minimum of the function.

#### 2.3 Curve sketching.

- Vertical asymptotes.  $x = a$ :  $\lim_{x \rightarrow a^+} f(x) = \pm\infty$  and/or  $\lim_{x \rightarrow a^-} f(x) = \pm\infty$ .
- Horizontal asymptotes.  $y = b$ :  $\lim_{x \rightarrow \infty} f(x) = b$  and/or  $\lim_{x \rightarrow -\infty} f(x) = b$ .
- Intercepts with  $x$ -axis and  $y$ -axis.