

Sample Math 115 Final Exam Spring Semester 2005

The final is on May 17, Tuesday 4:30 – 7:00PM. Contact your instructor for the exam room schedule. Note that you have to bring a # 2 pencil to the exam. Calculators are allowed for all parts of the exam but some problems may ask for an exact value. The problems below, all multiple-choice with exactly one correct answer choice, are intended to be reasonably representative of what might appear on the actual exam, which will have 30 problems.

The final exam will cover

- Chapter 2: Sections 2.1, 2.2, 2.3, 2.4, 2.5, 2.6;
 - Chapter 3: Sections 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7;
 - Chapter 4: Sections 4.1, 4.2, 4.3, 4.4, 4.5;
 - Chapter 5: Sections 5.1, 5.2, 5.4, 5.5, 5.6;
 - Chapter 6: Sections 6.1, 6.2, 6.3, 6.4, 6.5
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1. Let $f(x) = \frac{x}{x^2+1}$ and $g(x) = \frac{1}{x}$. Then, $(g \circ f)(x)$ is

- (A) $\frac{x}{x^2+1}$ (B) $\frac{1}{x}$ (C) $x + \frac{1}{x}$ (D) x (E) None of the above

2. Suppose that $F(x) = f(x)^2 + 1$, $f(1) = 1$, and $f'(1) = 3$. Find $F'(1)$.

- (A) 3 (B) 4 (C) 5 (D) 6 (E) None of the above

3. The unit price p and the quantity x demanded are related by the demand equation $50 - p(x^2 + 1) = 0$. Find the revenue function $R = R(x)$.

- (A) $\frac{50x}{x^2+1}$ (B) $\frac{50}{x^2+1}$ (C) $\frac{x}{x^2+1}$ (D) $\frac{x^2+1}{50}$

- (E) None of the above

4. Find the marginal revenue for the revenue function found in Problem 3.

(A) $\frac{-100x}{(x^2 + 1)^2}$ (B) $\frac{1 - x^2}{(x^2 + 1)^2}$ (C) $\frac{x}{25}$ (D) $\frac{50(1 - x^2)}{(x^2 + 1)^2}$

(E) None of the above

5. Find $\frac{dy}{dx}$ in terms of x and y when x and y are related by the equation $x^{1/3} + y^{1/3} = 1$.

(A) $-\left(\frac{x}{y}\right)^{2/3}$ (B) $-\left(\frac{y}{x}\right)^{2/3}$ (C) $-\left(\frac{x}{y}\right)^{1/3}$ (D) $-\left(\frac{y}{x}\right)^{1/3}$

(E) None of the above

6. Find $\frac{dy}{dx}$ at point $(2, 2\sqrt{5})$ when x and y are related by the equation $y^2 - x^2 = 16$.

(A) $\frac{\sqrt{5}}{5}$ (B) $\sqrt{5}$ (C) $\frac{5}{\sqrt{5}}$ (D) $\frac{1}{5\sqrt{5}}$ (E) None of the above

7. Let $f(x) = \frac{\sqrt{x+1}}{x-2}$. The domain of f is

(A) $(-\infty, 2)$ and $(2, +\infty)$ (B) $(-\infty, 2]$ and $[2, +\infty)$ (C) $[-1, 2)$ and $(2, +\infty)$

(D) $[-1, +\infty)$ (E) None of the above

8. Let $f(x) = \ln(2 - x)$. The domain of f is

(A) $(-\infty, +\infty)$ (B) $(0, +\infty)$ (C) $(-\infty, 0)$

(D) $(-\infty, 2)$ (E) None of the above

9. Evaluate: $\lim_{x \rightarrow 3} (3x^2 - 4)$.

(A) 23 (B) 5 (C) 4

(D) The limit does not exist (E) None of the above

10. Evaluate $\lim_{x \rightarrow 5} \frac{x^2 - 2x - 15}{x - 5}$.

(A) 3 (B) 8 (C) 0

(D) The limit does not exist (E) None of the above

11. Evaluate $\lim_{x \rightarrow 1^+} \frac{x^2 - 1}{2x - 2}$.

(A) -1 (B) 0 (C) +1

(D) The limit does not exist (E) None of the above

12. Evaluate $\lim_{x \rightarrow 1^-} \frac{x^2-1}{2x-2}$.
 (A) -1 (B) 0 (C) $+1$
 (D) The limit does not exist (E) None of the above
13. Find the horizontal asymptotes of function $f(x) = \frac{x^2}{1+4x^2}$.
 (A) $y = 1$ (B) $y = \frac{1}{4}$ (C) $x = 1$
 (D) The function has no horizontal asymptotes (E) None of the above
14. Find the vertical asymptotes of function $f(x) = \frac{2+x}{(1-x)^2}$.
 (A) $x = -2$ (B) $x = 1$ (C) $y = 0$
 (D) The function has no vertical asymptotes (E) None of the above
15. The line tangent to $y = x^2 - 3x$ through the point $(1, -2)$ has equation
 (A) $y = x - 3$ (B) $y + 2 = (2x - 3)(x - 1)$ (C) $y = -x - 1$
 (D) $y - 2 = (2x - 3)(x - 1)$ (E) None of the above
16. Find an equation of the tangent line to the graph of $y = x \ln x$ at the point $(1, 0)$.
 (A) $y = x + 1$ (B) $y = x - 1$ (C) $y = (x + 1) \ln x$
 (D) $y = (x - 1) \ln x$ (E) None of the above
17. Find an equation of the tangent line to the graph of $y = \ln(x^2)$ at the point $(2, \ln 4)$.
 (A) $y = x + 2 - \ln 4$ (B) $y = \frac{2}{x}(x - 2) - \ln 4$ (C) $y = \frac{2}{x}(x - 2) + \ln 4$
 (D) $y = x - 2 + \ln 4$ (E) None of the above
18. Find an equation of the tangent line to the graph of $y = e^{2x-3}$ at the point $(\frac{3}{2}, 1)$.
 (A) $y = 2e^{2x-3}$ (B) $y = 2x - 4$ (C) $y = 2x - 2$
 (D) $y = 2e^{2x-3}(x - \frac{3}{2})$ (E) None of the above
19. Find an equation of the tangent line to the graph of $y = e^{-x^2}$ at the point $(1, 1/e)$.
 (A) $y = -\frac{2}{e}(x + 1) + \frac{1}{e}$ (B) $y = -\frac{2}{e}(x - 1) - \frac{1}{e}$ (C) $y = -\frac{2}{e}(x - 1) + \frac{1}{e}$
 (D) $y = -2xe^{-x^2}(x - 1) + \frac{1}{e}$ (E) None of the above

20. The absolute maximum value and the absolute minimum value of the function $f(x) = \frac{1}{2}x^2 - 2\sqrt{x}$ on $[0, 3]$ are

(A) absolute min. value: $-\frac{3}{2}$; absolute max. value: $\frac{9}{2} - 2\sqrt{3}$

(B) absolute min. value: 0; absolute max. value: 3

(C) absolute min. value: 0; no absolute max. value

(D) no absolute min. value ; absolute max. value: 3

(E) None of the above

21. Find the absolute maximum value and the absolute minimum value, if any, of the function $f(x) = \frac{1}{1+x^2}$.

(A) absolute min. value: 0; absolute max. value: 1

(B) absolute min. value: 0; no absolute max. value

(C) no absolute min. value ; absolute max. value: 1

(D) no absolute min. value ; no absolute max. value

(E) None of the above

22. Find the absolute extrema of function $f(t) = te^{-t}$.

(A) absolute min. value: 0; absolute max. value: $\frac{1}{e}$

(B) absolute min. value: 0; no absolute max. value

(C) no absolute min. value ; absolute max. value: $\frac{1}{e}$

(D) no absolute min. value ; no absolute max. value

(E) None of the above

23. Find the absolute extrema of the function $f(t) = \frac{\ln t}{t}$ on $[1, 2]$.

(A) absolute min. value: 0; absolute max. value: $\frac{\ln 2}{2}$

(B) absolute min. value: 0; absolute max. value: $\frac{1}{e}$

(C) absolute min. value: 1; absolute max. value: 2

(D) absolute min. value: 0; absolute max. value: e

(E) None of the above

24. Let $f(x) = \frac{1}{3}x^3 - x^2 + x - 6$. Determine the intervals where the function is increasing and where it is decreasing.

(A) increasing on $(-\infty, 1)$ and on $(1, \infty)$

(B) increasing on $(-\infty, 1)$ and decreasing on $(1, \infty)$

(C) decreasing on $(-\infty, 1)$ and increasing on $(1, \infty)$

(D) decreasing on $(-\infty, 1)$ and on $(1, \infty)$

(E) None of the above

25. Let the function f be defined in Problem 24. Find the intervals where f is concave upward and where it is concave downward.

(A) concave upward on $(-\infty, 1)$ and on $(1, \infty)$

(B) concave upward on $(-\infty, 1)$ and downward on $(1, \infty)$

(C) concave downward on $(-\infty, 1)$ and upward on $(1, \infty)$

(D) Concave downward on $(-\infty, 1)$ and on $(1, \infty)$

(E) None of the above

26. Let the function f be defined in Problem 24. Find the inflection points, if any.

(A) $(x, y) = (1, f(1))$ (B) $(x, y) = (\frac{1}{2}, f(\frac{1}{2}))$ (C) $(x, y) = (0, f(0))$

(D) No inflection points (E) None of the above

27. Let $f(x) = e^{-x^2}$. Determine the intervals where the function is increasing and where it is decreasing.

(A) increasing on $(-\infty, 0)$ and on $(0, \infty)$

(B) increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$

(C) decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$

(D) decreasing on $(-\infty, 0)$ and on $(0, \infty)$

(E) None of the above

28. Let the function f be defined in Problem 27. Find the relative extrema of f .

(A) relative min. value: 0; relative max. value: 1

(B) no relative min. value ; relative max. value: 1

(C) relative min. value: 0; no relative max. value

(D) no relative min. value ; no relative max. value

(E) None of the above

29. Let the function f be defined in Problem 27. Find the intervals where f is concave upward and where it is concave downward.

- (A) concave upward on $(-\infty, 0)$ and on $(0, \infty)$
- (B) concave downward on $(-\infty, 0)$ and on $(0, \infty)$
- (C) concave upward on $(-\infty, -\frac{1}{\sqrt{2}})$ and on $(\frac{1}{\sqrt{2}}, \infty)$; concave downward on $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$
- (D) concave downward on $(-\infty, -\frac{1}{\sqrt{2}})$ and on $(\frac{1}{\sqrt{2}}, \infty)$; concave upward on $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$
- (E) None of the above

30. Let the function f be defined in Problem 27. Find the inflection points, if any.

- (A) $(x, y) = (0, f(0))$ (B) $(x, y) = (-\frac{1}{\sqrt{2}}, f(-\frac{1}{\sqrt{2}}))$ and $(x, y) = (\frac{1}{\sqrt{2}}, f(\frac{1}{\sqrt{2}}))$
- (C) $(x, y) = (-\frac{1}{\sqrt{2}}, f(-\frac{1}{\sqrt{2}}))$ (D) $(x, y) = (\frac{1}{\sqrt{2}}, f(\frac{1}{\sqrt{2}}))$ (E) None of the above

31. Let $f(x) = x \ln x$. Determine the intervals where the function is increasing and where it is decreasing.

- (A) increasing on $(-\infty, \frac{1}{e})$ and decreasing on $(\frac{1}{e}, \infty)$
- (B) decreasing on $(-\infty, \frac{1}{e})$ and increasing on $(\frac{1}{e}, \infty)$
- (C) increasing on $(0, \frac{1}{e})$ and decreasing on $(\frac{1}{e}, \infty)$
- (D) decreasing on $(0, \frac{1}{e})$ and increasing on $(\frac{1}{e}, \infty)$
- (E) None of the above

32. Suppose that f is defined in Problem 31. Determine the intervals of concavity for the function.

- (A) concave upward on $(0, \infty)$
- (B) concave downward on $(0, \infty)$
- (C) concave upward on $(0, \frac{1}{e})$; concave downward on $(\frac{1}{e}, \infty)$
- (D) concave downward on $(0, \frac{1}{e})$; concave upward on $(\frac{1}{e}, \infty)$
- (E) None of the above

33. Suppose that f is defined in Problem 31. Find the inflection points, if any.

(A) $(x, y) = (\frac{1}{e}, f(\frac{1}{e}))$ (B) $(x, y) = (1, f(1))$ (C) $(x, y) = (e, f(e))$

(D) No inflection points (E) None of the above

34. Find the derivative of function $y = x^{\ln x}$. (Hint: use logarithmic differentiation.)

(A) $y' = (\ln x)^2$ (B) $y' = \frac{2 \ln x}{x} x^{\ln x}$ (C) $y' = x^{\ln x}$

(D) the derivative does not exist (E) None of the above

35. Find the derivative of function $y = 10^x$. (Hint: use logarithmic differentiation.)

(A) $y' = 10^x \ln 10$ (B) $y' = 10^x$ (C) $y' = 10^x \ln e$

(D) the derivative does not exist (E) None of the above

36. An open box is to be made from a square sheet of tin measuring 12 inches \times 12 inches by cutting out a square of side x inches from each corner of the sheet and folding up the four resulting flaps. To maximize the volume of the box, take $x =$

(A) 1 (B) 2 (C) 3

(D) 4 (E) None of the above

37. A rectangular box is to have a square base and a volume of $20ft^3$. If the material for the base costs 30 cents/square, the material for the four sides costs 10 cents/square, and the material for the top costs 20 cents/square, determine the dimensions of the box that can be constructed at minimum cost. (See Fig. 1.)

(A) $x \times x \times h = 1 \times 1 \times 20$ (B) $x \times x \times h = 2 \times 2 \times 5$ (C) $x \times x \times h = 2.5 \times 2.5 \times 3.2$

(D) $x \times x \times h = 3 \times 3 \times 2.22$ (E) None of the above

38. Postal regulations specify that a parcel sent by parcel post may have a combined length and girth of no more than 108 inches. Find the dimensions of the cylindrical package of greatest volume that may be sent through the mail. (In the answers, r is the radius and l is the length.)

(A) $r \times l = \frac{35}{\pi} \times 37$ (B) $r \times l = \frac{36}{\pi} \times 36$ (C) $r \times l = \frac{37}{\pi} \times 35$

(D) $r \times l = \frac{38}{\pi} \times 34$ (E) None of the above

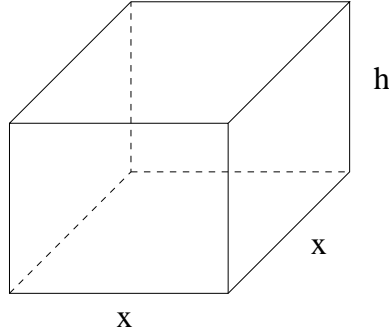


Figure 1: Problem 37.

39. It costs an artist $1000 + 5x$ dollars to produce x signed prints of one of her drawings. The price at which x prints will sell is $400/\sqrt{x}$ dollars per print. How many prints should she make in order to maximize her profit ?

- (A) 1200 (B) 1400 (C) 1600 (D) 1800
 (E) None of the above

40. The differential of function $f(x) = 1000$ is

- (A) 1000 (B) $1000dx$ (C) 0 (D) dx
 (E) None of the above

41. Use differentials to estimate the change in $\sqrt{x^2 + 5}$ when x increases from 2 to 2.123.

- (A) 0.083 (B) 0.082 (C) 0.081 (D) 0.080
 (E) None of the above

42. The velocity of a car (in feet/second) t seconds after starting from rest is given by the function

$$f(t) = 2\sqrt{t} \quad (0 \leq t \leq 30).$$

Find the car's position at any time t .

- (A) $\frac{4}{3}t^{3/2} + C$ (B) $\frac{4}{3}t^{3/2}$ (C) $\frac{4}{3}t^{1/2} + C$
 (D) $\frac{4}{3}t^{1/2}$ (E) None of the above

43. Evaluate $\int (\sqrt{x} - 2e^x) dx$.

- (A) $\frac{2}{3}x^{3/2} - 2e^x$ (B) $\frac{2}{3}x^{3/2} - 2e^x + C$ (C) $\frac{3}{2}x^{2/3} - 2e^x$
 (D) $\frac{3}{2}x^{2/3} - 2e^x + C$ (E) None of the above

44. Calculate $\int_1^8 \left(4x^{1/3} + \frac{8}{x^2}\right) dx$.

(A) 49 (B) 50 (C) 51

(D) 52 (E) None of the above

45. Evaluate $\int_0^3 |1 - x| dx$.

(A) 3/2 (B) 5/2 (C) 7/2 (D) 9/2

(E) None of the above

46. Evaluate $\int x e^{-x^2} dx$.

(A) $-e^{-x^2} + C$ (B) $-\frac{1}{2}e^{-x} + C$ (C) $(1 - 2x^2)e^{-x^2} + C$ (D) $-\frac{1}{2}e^{-x^2} + C$

(E) None of the above

47. Find the area of the region under the graph of function $f(x) = x^2$ on the interval $[0, 1]$.

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$

(E) None of the above

48. Find the area of the region under the graph of $y = x^2 + 1$ from $x = -1$ to $x = 2$.

(A) 4 (B) 5 (C) 6 (D) 7

(E) None of the above