Find equations of both tangent lines to the ellipse \( x^2 + 4y^2 = 36 \) that pass through the point \((12,3)\).

* Observe that \((12,3)\) is not on the ellipse.

**Step 1. Implicit Differentiation.**

\[
\frac{d}{dx}(x^2 + 4y^2) = \frac{d}{dx}(36) \quad \Rightarrow \quad 2x + 8y \frac{dy}{dx} = 0
\]

\[
\Rightarrow \quad \frac{dy}{dx} = \frac{-x}{4y}
\]

**Step 2. This is the tricky part...**

Call either of the points where the tangent line touches the ellipse \((x_1,y_1)\). Then we know that at this point, the slope of the tangent line is \(\frac{-x_1}{4y_1}\).

On the other hand, the tangent line also has to pass through \((12,3)\), so the basic slope formula gives \(\frac{3-y_1}{12-x_1}\).

Equating these quantities, we get:

\[
-\frac{x_1}{4y_1} = \frac{3-y_1}{12-x_1} \quad \Rightarrow \quad -x_1(12-x_1) = 4y_1(3-y_1). \tag{1}
\]

Since \((x_1,y_1)\) is on the ellipse, we know that \(x_1^2 + 4y_1^2 = 36\). \tag{2}

**Step 3.** We now have 2 equations in 2 unknowns. It's not easy, but you can now solve for \((x_1,y_1)\) and get the tangent lines.