Power Series

A power series has the form \( \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + \ldots + c_n x^n + \ldots \)

Where \( x \) is a variable, the constants \( c_n \)'s are coefficients.

Given a \( x \) value, the power series is a series of numbers, which we have seen in previous sections.

For different \( x \) values, the power series could be convergent or divergent.

If the power series is convergent for a set of \( x \) values, then it is a function over these \( x \) values.

\[
\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1 (x-a) + c_2 (x-a)^2 + \ldots + c_n (x-a)^n + \ldots
\]
is a power series centered at \( x = a \).

* Power Series Convergence Theorem

Only three possibilities for \( \sum_{n=0}^{\infty} c_n (x-a)^n \):

1. The series converges only for \( x = a \) \((R=0)\)
2. The series converges for all \( x \) \((R=\infty)\)
3. There is a number \( R > 0 \) such that the series converges for \( |x-a| < R \) and diverges for \( |x-a| > R \)

* Using Ratio Test to find \( R \).

\[
\lim_{n \to \infty} \left| \frac{c_{n+1}}{c_n} \right| = L,
\]
then \( R = \frac{1}{L} \)

\[\xrightarrow{\text{diverges}} \quad \xrightarrow{\text{converges}} \quad \xrightarrow{\text{diverges}}\]
Endpoints: Need checking!