1. Differentiation and Integration of Power Series

If \( f(x) = \sum_{n=0}^{\infty} C_n (x-a)^n \) has radius of convergence \( R > 0 \), then \( f(x) \)
is differentiable on the interval \((a-R, a+R)\).

(i) \( f'(x) = \sum_{n=1}^{\infty} n C_n (x-a)^{n-1} \) (the radii of convergence is still \( R \))

(ii) \( \int f(x) \, dx = \text{Constant} + \sum_{n=0}^{\infty} \frac{C_n}{n+1} (x-a)^{n+1} \) (the radii of convergence is still \( R \))

2. \( \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \) for \( x \) values in \((-1, 1)\)

function represented as power series

\( \frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty} (-x)^n = \sum_{n=0}^{\infty} (-1)^n x^n \), \(-1<x<1\)

\( \frac{1}{1-x^2} = \sum_{n=0}^{\infty} (x^n)^n \), \(-1<x<1\) so \( x \in (-1, 1) \)

How about \( \ln(1-x) \), \( \frac{1}{1-x^2} \), ...?

Use differentiation or Integration to convert the function into \( \frac{1}{1-x} \)

Then Anti-operate (Integration or differentiation) power series.

This is the strategy to represent function into power series.

Don't forget to label the radii and interval of convergence.