Problem 1. (4 points) The contour plot for the function $f = f(x, y)$ is given below.

Determine whether the following partial derivatives are positive or negative at $P$:

A) $f_x > 0$  
B) $f_y > 0$  
C) $f_{xx} > 0$  
D) $f_{yx} < 0$

Problem 2. (3 points)

Find the linear approximation of the function $f(x, y) = \sqrt{x^2 + 4y^2}$ at $(3, 2)$ and use it to approximate the number $\sqrt{3.05^2 + 4 \cdot 2.05^2}$.

\[
f(x, y) \approx f(3, 2) + f_x(3, 2)(x-3) + f_y(3, 2)(y-2)
\]

\[
f_x(x, y) = \frac{x}{\sqrt{x^2 + 4y^2}}, \quad f_y(x, y) = \frac{4y}{\sqrt{x^2 + 4y^2}}\]

so, the linearization is

\[
f(x, y) \approx 5 + \frac{3}{\sqrt{10}}(x-3) + \frac{8}{5}(y-2)
\]

and

\[
\sqrt{3.05^2 + 4 \cdot 2.05^2} = f(3.05, 2.05) \approx 5 + \frac{3}{\sqrt{10}} \cdot 0.05 + \frac{8}{5} \cdot 0.05 = 5.11
\]
Problem 3. (3 points) The radius of a cylinder is increasing at the rate \(2 \frac{m}{s}\). At what rate should the height be decreasing in order for the volume of the cylinder to stay constant when the radius is 1 m and the height is 2 m? Recall that the volume of a cylinder with height \(h\) and radius \(r\) is given by \(V(r, h) = \pi r^2 h\).

Since the volume of the cylinder stays constant we have that
\[
\frac{d}{dt} (V(r,h)) = 0.
\]

On the other hand,
\[
\frac{d}{dt} (V(r,h)) = \frac{dV}{dr} \cdot \frac{dr}{dt} + \frac{dV}{dh} \cdot \frac{dh}{dt}
\]

so,
\[
2\pi rh \cdot 2 + \pi r^2 \cdot \frac{dh}{dt} = 0
\]

when \(r=1\) and \(h=2\) we get
\[
\frac{dh}{dt} = -8 \left( \frac{m}{s} \right).
\]