Problem 1. (4 points) Evaluate the integral

$$\int \int_{R} \sin(9x^2 + 4y^2) \, dA,$$

where $R$ is the region in the first quadrant bounded by the ellipse $9x^2 + 4y^2 = 1$, by making an appropriate change of variables. \textit{Hint: use the change of variables} $u = 3x$, $v = 2y$. \textit{You can use the fact that under this transformation the region} $R$ \textit{is mapped into the part of the unit circle} $u^2 + v^2 = 1$ \textit{in the uv-plane which lies in the first quadrant without showing it.}

$$u = 3x, \quad v = 2y \quad \Rightarrow \quad x = \frac{u}{3}, \quad y = \frac{v}{2}$$

The Jacobian of this transformation is

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{vmatrix} = \frac{1}{6}.$$

So,

$$\int \int_{R} \sin(9x^2 + 4y^2) \, dA = \int \int_{S} \sin((u^2 + v^2)^{\frac{1}{2}}) \, ududv = \frac{1}{6} \int_{0}^{\pi/2} \int_{0}^{1} \sin(u^2 + v^2) \, rdrd\theta$$

$$S = \{ u^2 + v^2 \leq 1, \, u, \, v > 0 \}$$

$$= \frac{\pi}{24} (-\cos(1)) \bigg|_{0}^{1} = \frac{\pi}{24} (1 - \cos(1))$$

Problem 2. (2 points) Use the geometric interpretation to evaluate the integral $\int_{C} 1 \, ds$ where $C$ consists of the line segments from $(1, 0)$ to $(1, 1)$ and from $(1, 1)$ to $(0, 2)$. Explain your answer.

$$\int_{C} 1 \, ds = \text{Shaded area}$$

$$= 1 \cdot 1 + 1 \cdot \sqrt{2} = 1 + \sqrt{2}$$
Problem 3. (4 points) Evaluate the line integral \( \int_C zdx + xdy + ydz \), where \( C \) can be parameterized as \( x = t^2, \ y = t^3, \ z = t^2, \ 0 \leq t \leq 1 \).

\[
\int_C zdx + xdy + ydz = \int_0^1 \left[ t^2 \cdot x'(t) \ dt + t^2 \cdot y'(t) \ dt + t^2 \cdot z'(t) \ dt \right]
\]

\[
= \int_0^1 \left[ 2t + 2t^2 + 3t^2 \right] \ dt
\]

\[
= \left( \frac{1}{2} t^2 + 2t^3 \right) \bigg|_0^1 = \frac{1}{2} + 1 = \frac{3}{2}.
\]