Problem 1. True or False:

A) A function can have 3 different horizontal asymptotes.

[False]

(If it was true then as x approaches +\infty or -\infty
no would have at least 2 different limits which is
not possible since the limit must be unique.)

B) Suppose that for some function f we know that f(2) < 0 and f(3) > 0 then there exist
and number c from the interval (2, 3) such that f(c) = 0.

[False]

Note: The IVT holds only
for continuous functions

C) If f(x) > 0 for all x's then \( \lim_{x \to 0} f(x) > 0 \).

[False]

Consider \( f(x) = \begin{cases} x^2, & x \neq 0, \\ 1, & x = 0. \end{cases} \)

Clearly, \( f(x) > 0 \) for all x's. However, \( \lim_{x \to 0} f(x) = 0 \).

Moreover, the limit doesn't have to exist at all.

Problem 2. Find \( \lim_{x \to +\infty} e^{-x} \sin x \). Provide explanation. Hint: Squeeze Theorem.

Since \( -1 \leq \sin x \leq 1 \), we have that

\[ -e^{-x} \leq e^{-x} \sin x \leq e^{-x} \]

\[ \lim_{x \to \infty} e^{-x} = 0 \]

By the Squeeze Theorem \( \lim_{x \to \infty} e^{-x} \sin x = 0 \).
**Problem 3.** The figure shows the graphs of $f$, $f'$, and $f''$. Identify each curve.

We conclude that $f = a$, $f' = c$, $f'' = b$.

**Problem 4.** Find numbers $a$ and $b$ such that

$$\lim_{x \to 0} \frac{\sqrt{ax + b} - 2}{x} = 1$$

\[
\lim_{x \to 0} \frac{\sqrt{ax + b} - 2}{x} = \lim_{x \to 0} \frac{ax + b - 4}{x(\sqrt{ax + b} + 2)} \quad \text{[to get rid of $x$ in the denominator we have to choose $b = 4$]}
\]

\[
\lim_{x \to 0} \frac{ax}{(ax + 4) + 2} = \lim_{x \to 0} \frac{a}{\sqrt{ax + 4} + 2} = \frac{a}{4} \implies a = 4
\]