Math 115
Fall 2012
Answers to the Exam 3 Review Problems

1. (a) concave up everywhere; (b) relative minimum at $x = 1$. (c)

![Graph](image1)

2. (a) Vertical asymptotes: $x = b, x = -b$. (b) Horizontal asymptote: $y = -1$.

3. (a) Vertical asymptote: $x = b$. (b) Horizontal asymptote: $x = 1$. (c) No relative extrema. (d) Concave up if $x > b$. Concave down if $x < b$. (d) Looks like the graph of $y = \frac{1}{x}$ but moved over to match the asymptotes

4. (a) Relative min at $f(100) = e^{100} - 100e^{100}$. (b) It is everywhere concave up ($f''(x) = e^x > 0$ for every $x$). (c)

![Graph](image2)

5. Since $y = \ln a + \ln x$, it looks like an normal log function only shifted upward by $\ln a$.

6. The graph should vaguely resemble:

![Graph](image3)
7. Here’s what’s true: (iii), (iv), (v). There is a serious typo: (viii) was supposed to read \( f''(1) > 0 \) and that would be correct. You have no way to determine the exact value of \( f''(1) \).

8. The critical point is \( x = 0 \). Since it is the only critical point and \( f(0) \) is a relative min, then \( f(0) = -1/2 \) is an absolute min. The absolute max is \( f[2] \approx 3.619 \times 10^{86} \).

9. In this interval, the maximum value is 80,000, obtained when \( x = -20 \) and \( x = 20 \); the minimum value is \( -10,000 \), obtained when \( x = -10 \) and \( x = 10 \).

10. (a) The dimensions that maximize the volume are \( x = 10 \sqrt{3/2} \approx 8.165 \), \( y = 10 \sqrt{3/2 - 5 \sqrt{3/2}} \approx 8.165 \). (In fact, \( x = y \). Try squaring both to see what you get.) The maximum volume is about 544.33.

(b) \( V'(x) < 0 \) for any \( x > 0 \), so the critical point gives us a relative maximum. Since it is the only critical point in the interval \((0, \infty)\), it is an absolute maximum.

11. (a) \( p = $1.75 \), \( R = $612.50 \). (b) Since \( R''(.75) = -1 \), we have a relative maximum. Since there’s only one critical point, it’s an absolute maximum.

12. (a) The dimensions are \( x = 10 \), \( y = 25 \). \( A = 250 \).

(b) We can do this the way we did in #10 and #11, but there’s another way also: \( 0 \leq x \leq 100 \), so we can check the endpoints: \( A(0) = 0 \), \( A(10) = 250 \), \( A(100) = -20,000 \), so we have an absolute maximum.

13. (a) \( 2e^{2x} \). (b) any of the following (or something similar): \( 2^{12x-3} \frac{9x}{8} ; \frac{2^{12x} 3^{9x}}{8} ; \frac{6^{9x} 3^x}{8} \), etc.

14. \( x = \pm 2 \)

15. \( x = 1,000 \)

16. (a) \( f'(x) = (2x - 3x^2)e^{x^2-x^3} \)

(b) \( g'(x) = \frac{2x - 3x^2}{x^2 - x^3} \)

(c) \( h'(x) = \frac{1}{x} + 1. \)

(d) \( j'(x) = -\frac{2x}{(x^2 + 1)^2} e^{1/(x^2+1)} \)

17. \( f'(x) = (x-1)(x+1)^2(x-2)^3 \left( \frac{1}{x-1} + \frac{2}{x+1} + \frac{3}{x-2} + \frac{4}{x+2} \right) \)

18. \( k = \frac{10 \ln 2}{\ln 3} \approx 6.3 \) years.

19. \( t = \frac{5,730 \ln 3}{\ln 3} \approx 9081 \)

20. 100

21. \( k \approx -0.13 \).