

**THE 26th ANNUAL MATHEMATICS PRIZE
COMPETITION
(JUNIOR)**

Department of Mathematics

University of Kansas

March 31, 2008

Instructions

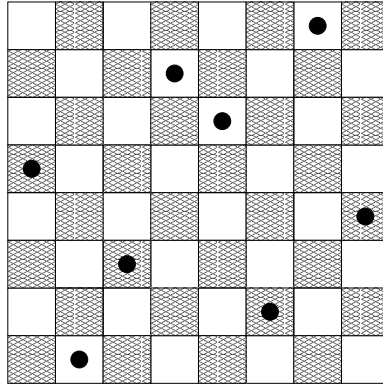
1. **DO NOT** write your name on your solutions; use your student number. Write your student number on each solution that you submit.
2. Start **each** solution on a separate sheet. If a solution takes more than one page, please number the pages and staple them together. (*Solutions of **different** problems should **not** be stapled together.*)
3. You need not hand in your scratch work.
4. You may keep this page.
5. No calculators, books, notes, or other aids are permitted.
6. Each of the 6 problems is worth 10 points. For full credit, you must explain your answers. If a problem has more than one part, the parts do not necessarily have equal weight. Partial credit may be given for significant progress towards a solution.
7. Winners will be notified by mail.
8. Have fun!

- J1.** If $f(x)$ is a function and n is a positive integer, let $F_n(x)$ be the function obtained by iterating $f(x)$ precisely n times. That is,

$$F_1(x) = f(x), \quad F_2(x) = f(f(x)), \quad F_3(x) = f(f(f(x))),$$

et cetera. If $f(x) = 1/(1 - x)$, evaluate $F_{2008}(2008)$.

- J2.** (a) How many ways can you place eight rooks on a chessboard so that no rook attacks any other rook? An example of one such way is shown in the diagram below.



(b) How many ways can you place the rooks so that no rook attacks any other rook and *every* rook is on a white square?

(c) How many ways can you place the rooks so that no rook attacks any other rook and *exactly one* rook is on a white square?

J3. Evaluate $\sum_{n=0}^{\infty} \frac{n^2 + 1}{2^n}$.

- J4.** Suppose that every student at KU counts the number of other students with whom he or she has taken a class. Prove that some pair of students will get the same number.

J5. Let $x > 0$ be a real number. Which is greater, $\frac{x}{x+1}$ or $\ln(1 + x)$? Prove your answer.

- J6.** The island of Trufalsia is inhabited by knights and knaves. The knights always tell the truth, and the knaves always lie. On a visit to Trufalsia, you encounter three inhabitants named Xerxes, Yvette and Zeno. The conversation goes like this:

Xerxes: All three of us are knaves.

Yvette: Actually, Zeno is a knight.

Which of these three are knights and which are knaves?