Ranking All The Candidates

Once we’ve chosen a voting method and a declared a winner, how do we know who finished second? Third? Last?

Why do this? For example:

- Electing officers: Winner becomes president, second-place finisher becomes vice-president, . . .
- Draft choices: Take the highest-ranked player available

Each voting method we’ve studied can be used as an extended ranking method.
The Extended Plurality Method

<table>
<thead>
<tr>
<th>Number of Voters</th>
<th>14</th>
<th>10</th>
<th>8</th>
<th>4</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st choice</td>
<td>A</td>
<td>C</td>
<td>D</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>2nd choice</td>
<td>B</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>3rd choice</td>
<td>C</td>
<td>D</td>
<td>B</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>4th choice</td>
<td>D</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Candidate</th>
<th>Votes</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alesha</td>
<td>14</td>
<td>1st place</td>
</tr>
<tr>
<td>Boris</td>
<td>4</td>
<td>4th place</td>
</tr>
<tr>
<td>Carmen</td>
<td>11</td>
<td>2nd place</td>
</tr>
<tr>
<td>Dave</td>
<td>8</td>
<td>3rd place</td>
</tr>
</tbody>
</table>
The Extended Borda Count

<table>
<thead>
<tr>
<th>Number of Voters</th>
<th>14</th>
<th>10</th>
<th>8</th>
<th>4</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st choice</td>
<td>A</td>
<td>C</td>
<td>D</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>2nd choice</td>
<td>B</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>3rd choice</td>
<td>C</td>
<td>D</td>
<td>B</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>4th choice</td>
<td>D</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Candidate</th>
<th>Borda Count</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alesha</td>
<td>79</td>
<td>4th place</td>
</tr>
<tr>
<td>Boris</td>
<td>106</td>
<td>1st place</td>
</tr>
<tr>
<td>Carmen</td>
<td>104</td>
<td>2nd place</td>
</tr>
<tr>
<td>Dave</td>
<td>81</td>
<td>3rd place</td>
</tr>
</tbody>
</table>
## The Extended Method of Pairwise Comparisons

### Number of Voters

<table>
<thead>
<tr>
<th>Number of Voters</th>
<th>14</th>
<th>10</th>
<th>8</th>
<th>4</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st choice</td>
<td>A</td>
<td>C</td>
<td>D</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>2nd choice</td>
<td>B</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>3rd choice</td>
<td>C</td>
<td>D</td>
<td>B</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>4th choice</td>
<td>D</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
</tbody>
</table>

### Candidate Comparisons

<table>
<thead>
<tr>
<th>Candidate</th>
<th>Would beat</th>
<th>Comparisons won</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alesha</td>
<td>Nobody</td>
<td>0</td>
<td>4th place</td>
</tr>
<tr>
<td>Boris</td>
<td>A, D</td>
<td>2</td>
<td>2nd place</td>
</tr>
<tr>
<td>Carmen</td>
<td>A, B, D</td>
<td>3</td>
<td>1st place</td>
</tr>
<tr>
<td>Dave</td>
<td>A</td>
<td>1</td>
<td>3rd place</td>
</tr>
</tbody>
</table>
# The Extended PWE Method

## Number of Voters

<table>
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<tr>
<th>Number of Voters</th>
<th>14</th>
<th>10</th>
<th>8</th>
<th>4</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st choice</td>
<td>A</td>
<td>C</td>
<td>D</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>2nd choice</td>
<td>B</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>3rd choice</td>
<td>C</td>
<td>D</td>
<td>B</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>4th choice</td>
<td>D</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
</tbody>
</table>

## Candidate Elimination

<table>
<thead>
<tr>
<th>Candidate</th>
<th>When eliminated</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alesha</td>
<td>Third round</td>
<td>2nd place</td>
</tr>
<tr>
<td>Boris</td>
<td>First round</td>
<td>4th place</td>
</tr>
<tr>
<td>Carmen</td>
<td>Second round</td>
<td>3rd place</td>
</tr>
<tr>
<td>Dave</td>
<td>Never</td>
<td>1st place</td>
</tr>
</tbody>
</table>
Comparing Extended Ranking Methods

<table>
<thead>
<tr>
<th>Number of Voters</th>
<th>14</th>
<th>10</th>
<th>8</th>
<th>4</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st choice</td>
<td>A</td>
<td>C</td>
<td>D</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>2nd choice</td>
<td>B</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>3rd choice</td>
<td>C</td>
<td>D</td>
<td>B</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>4th choice</td>
<td>D</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Candidate</th>
<th>Plurality</th>
<th>Borda</th>
<th>PComp</th>
<th>PWE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alesha</td>
<td>1st</td>
<td>4th</td>
<td>4th</td>
<td>2nd</td>
</tr>
<tr>
<td>Boris</td>
<td>4th</td>
<td>1st</td>
<td>2nd</td>
<td>4th</td>
</tr>
<tr>
<td>Carmen</td>
<td>2nd</td>
<td>2nd</td>
<td>1st</td>
<td>3rd</td>
</tr>
<tr>
<td>Dave</td>
<td>3rd</td>
<td>3rd</td>
<td>3rd</td>
<td>1st</td>
</tr>
</tbody>
</table>
There is another approach to turn any voting method into a ranking method, using the idea of recursion.

We are skipping this material (pp. 24–26 of Tannenbaum); you are not responsible for it on the homework or tests.
Which voting method is best?

There is no purely mathematical answer to this question.

**Arrow’s Theorem:** There is no voting method that always satisfies all four voting criteria – Majority, Condorcet, Monotonicity and IIA.

So, the answer depends which fairness criteria you think are the most important.

- Be skeptical of claims that “the vast majority of mathematical experts on voting theory believe that X is the best method.”
Advocates of Various Voting Methods

We’re going to look at the way that different voting methods are portrayed by their advocates.

Disclaimer: I am not responsible for the content of external websites and neither endorse or oppose any non-mathematical opinions thereon.
The Borda count

**Prof. Don Saari**
Prominent mathematician/physicist; **advocate of Borda Count**

▶ Fun short read: **Talking about voting with fourth graders**

▶ Sample article: **“The optimal ranking is the Borda count”**

Despite the title, what Saari proves that **if you are going to assign points for places**, then the Borda count is the only way that, e.g., cannot rank the Condorcet winner last.

So, this article says nothing about comparing Borda with PWE or Pairwise Comps
CIVS (Condorcet Internet Voting Project)
Advocate of Method of Pairwise Comparisons

Criticizes PWE Method (= IRV = STV):

“...the Condorcet methods supported by CIVS are better than IRV at identifying consensus candidates. STV can elect a candidate even though a majority of voters would prefer someone else, and it has in real elections.”
Center for Voting and Democracy
Advocate of Plurality-with-Elimination Method (≈ IRV)

- **Compares** different voting methods by different fairness criteria — which criteria do they consider most/least important? (Remember, this is a matter of opinion.)

- **Criticizes** other voting methods, e.g.:

  *Does the method require the winner to have core support? We believe a winner should be at least one voter’s first choice, meaning they would receive more than 0% in current rules.*

- **Argues** that violations of Monotonicity are unlikely
Chapter 1 Summary

Voting systems: Plurality, Borda Count, Plurality-with-Elimination, Method of Pairwise Comparisons

Fairness criteria: Majority, Condorcet, Public-Enemy, Monotonicity, Independence-of-Irrelevant-Alternatives

Arrow’s Theorem: No voting method can always satisfy all fairness criteria.

Gibbard-Satterthwaite Theorem: No voting method is completely immune to strategic (insincere) voting.

Which voting system is best? This question has no single mathematical answer, but any debate needs to be informed by mathematics.