The Domino Problem

Take an $8 \times 8$ grid (64 squares).
The Domino Problem

Delete two opposite corners, leaving 62 squares.
Is it possible to place 31 dominoes on the grid, with every domino covering two squares, so that every square gets covered exactly once?
The answer: **No.**
To see why not, color the grid like a chessboard.
The Domino Problem

- The original grid had 32 white squares and 32 black squares.

- The two deleted squares were both black.

- So the new grid has 32 white squares, but only 30 black squares.

- Meanwhile, every domino has to cover one white and one black square.

- Therefore, it is impossible to cover the board with 31 dominoes!
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