Math 830 ABSTRACT ALGEBRA

QUIZ – XII

October 27 (Fri), 2006

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Line #: 17014.

ID : ____________________ Name : ______________________________

[I] (20pts) (1) Let $\mathbb{Q}\left[x,\ y,\ z\right]$ denote the polynomial ring with three variables.

Find the three <u>fundamental symmetric polynomials</u> in $\mathbb{Q}\left[x,\ y,\ z\right]$.

$\phi_1(x,\ y,\ z) =$

______________________________ ,

$\phi_2(x,\ y,\ z) =$

______________________________ ,

$\phi_3(x,\ y,\ z) =$

______________________________ .

(2) Let $\mathbb{Q}\left[x,\ y,\ z,\ w\right]$ denote the polynomial ring with four variables.

Find the four <u>fundamental symmetric polynomials</u> in $\mathbb{Q}\left[x,\ y,\ z,\ w\right]$.

$\phi_1(x,\ y,\ z,\ w) =$

______________________________ ,

$\phi_2(x,\ y,\ z,\ w) =$

______________________________ ,

$\phi_3(x,\ y,\ z,\ w) =$

______________________________ ,

$\phi_4(x,\ y,\ z,\ w) =$

______________________________ .

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([I] continued)

(3) Let $\mathbb{Q}\left[x_1, x_2, \cdots, x_n\right]$ denote the polynomial ring with n variables.

Find the k-th <u>fundamental symmetric polynomials</u> in $\mathbb{Q}\left[x_1, x_2, \cdots, x_n\right]$:

$$\phi_k\big(x_1,\ x_2,\ \cdots,\ x_n\big) \;=\; \sum \qquad \underline{\hspace{6cm}} \,.$$

[II] (10pts) Consider $R = \mathbb{Q}\left[x_1,\ x_2,\ x_3,\ x_4\right]$, and $G = S_4$-action on R.

Let

$$\sigma \;=\; \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}.$$

(1) $\Big(x_1x_2x_3 \;-\; x_4^2\Big)^{\sigma} \;=$ ______________________________ .

(2) $\Big[\Big(x_1 - x_2\Big)\Big(x_2 - x_3\Big)\Big(x_3 - x_4\Big)\Big]^{\sigma}$

$=$ ____ .

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([II] continued)

(3) $\left(x_1^3 - x_2^3 + x_3^3 - x_4^3\right)^\sigma =$

______________________ .

[III] (10pts) Let

$$A = \begin{bmatrix} x^2 & y^2 & z^2 \\ x & y & z \\ 1 & 1 & 1 \end{bmatrix}.$$

(1) Prove

$$\det A = \begin{vmatrix} x^2 & y^2 & z^2 \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix} = \left(x - y\right)\left(x - z\right)\left(y - z\right).$$

(2) Write out the matrix $A\,A^T$.

(3) Use (1–2) to prove

$$\begin{vmatrix} x^4 + y^4 + z^4 & x^3 + y^3 + z^3 & x^2 + y^2 + z^2 \\ x^3 + y^3 + z^3 & x^2 + y^2 + z^2 & x + y + z \\ x^2 + y^2 + z^2 & x + y + z & 1 + 1 + 1 \end{vmatrix}$$

$$= \left(x - y\right)^2 \left(x - z\right)^2 \left(y - z\right)^2.$$