Math 290 ELEMENTARY LINEAR ALGEBRA

SOLUTION FOR QUIZ – I (01/24)

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[I] (20pts) Consider the system
$$\begin{aligned} ax + by &= e, \\ cx + dy &= f \end{aligned} \qquad \left(a \neq 0\right).$$

(1) Multiply $-\dfrac{c}{a}$ to the first equation and then add it to the second equation.

The resulting equation is
$$\left(d - \frac{\boxed{bc}}{a}\right) y = f - \frac{\boxed{ce}}{a}.$$

(2) In the identity obtained in (1), multiply a to the both sides:
$$\left(ad - bc\right) y = af - ce.$$

Thus, <u>assuming $ad - bc \neq 0$</u>, we have
$$y = \frac{\boxed{af - ce}}{ad - bc}.$$

(3) From the result of (2) and the first equation $ax + by = e$, we have
$$\begin{aligned} x = \frac{1}{a}\left(e - by\right) &= \frac{1}{a}\left(e - \frac{b \cdot \left(\boxed{af - ce}\right)}{ad - bc}\right) \\ &= \frac{1}{a} \, \frac{e\left(ad - bc\right) - b\left(af - ce\right)}{ad - bc} \\ &= \frac{1}{a} \, \frac{ade - bce - abf + bce}{ad - bc} \\ &= \frac{1}{a} \, \frac{ade - abf}{ad - bc} \\ &= \frac{1}{a} \, \frac{a\left(de - bf\right)}{ad - bc} = \frac{\boxed{de - bf}}{ad - bc}. \end{aligned}$$

Hence
$$\left(x, \ y\right) = \left(\frac{de - bf}{ad - bc}, \ \frac{af - ce}{ad - bc}\right).$$

(4) When $ad - bc = 0$, the system of linear equations

$$\begin{aligned} ax + by &= e, \\ cx + dy &= f \end{aligned}$$

has either no solution or infinitely many solutions. Indeed, the system

$$\begin{aligned} 2x + y &= 1, \\ 10x + 5y &= 1 \end{aligned}$$

has no solution, whereas the system

$$\begin{aligned} 2x + y &= 1, \\ 10x + 5y &= 5 \end{aligned}$$

has infinitely many solutions $\left(x,\ y\right) = \left(t,\ 1 - 2t\right)$.