

**Math 290 ELEMENTARY LINEAR ALGEBRA**  
**QUIZ – II (TAKE HOME)**

January 29 (Tue), 2008

**Due Date:** January 31 (Thu), 2008

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**Line #:** 74449 / 82588 .

**ID :** \_\_\_\_\_

**Name :** \_\_\_\_\_

[I] (10pts) Let

$$\left\{ \begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = 0, \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = 0, \\ \vdots \quad \quad \quad \vdots \quad \quad \quad \vdots \quad \quad \quad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = 0 \end{array} \right.$$

be a system of linear equations (where  $a_{ij}$  are scalars), consisting of

$\boxed{m}$  equations, and  $\boxed{n}$  variables (= unknowns).

(1) The constant term of each of the equations in the above system is 0.

Accordingly, the system of equations is called \_\_\_\_\_ .

(2) The trivial solution to the above system of equations is

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} \boxed{\phantom{0}} \\ \boxed{\phantom{0}} \\ \vdots \\ \boxed{\phantom{0}} \end{bmatrix} .$$

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([I] continued)

- (3) Find a sufficient condition for the above system of equations to have a non-trivial solution, in terms of  $m$  and  $n$  :

\_\_\_\_\_ .

$$(4) \quad \left\{ \begin{array}{l} x_1 + x_2 + x_3 + x_4 + x_5 = 0, \\ x_2 + x_3 + x_4 + x_5 = 0, \\ x_3 + x_4 + x_5 = 0, \\ x_4 + x_5 = 0 \end{array} \right. \quad \text{is} \quad \begin{array}{l} \square \text{ a homogeneous system} \\ \square \text{ an inhomogeneous system} \end{array}$$

of linear equations.

- (5) The system of equations in (4)

- has trivial solution only.
- has a non-trivial solution

$$(x_1, x_2, x_3, x_4, x_5) = ( \quad , \quad , \quad , \quad , \quad ) .$$

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[II] (10pts)

- (1)  $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$
- is in reduced row echelon form.
  - is in row echelon form, not in reduced row echelon form.
  - is not in row echelon form.

- (2)  $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$
- is in reduced row echelon form.
  - is in row echelon form, not in reduced row echelon form.
  - is not in row echelon form.

- (3)  $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$
- is in reduced row echelon form.
  - is in row echelon form, not in reduced row echelon form.
  - is not in row echelon form.

- (4)  $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
- is in reduced row echelon form.
  - is in row echelon form, not in reduced row echelon form.
  - is not in row echelon form.

- (5)  $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
- is in reduced row echelon form.
  - is in row echelon form, not in reduced row echelon form.
  - is not in row echelon form.

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([II] continued)

(6) Give a complete list of reduced row echelon form for  $3 \times 3$  matrices.

Use \* to denote an arbitrary entry .

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 & * \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}, \quad \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}, \quad \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix},$$

$$\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}, \quad \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}.$$