Math 290 ELEMENTARY LINEAR ALGEBRA

SOLUTION FOR QUIZ – III (01/29)

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[I] (8pts) (1) The augmented matrix for the system of linear equations

$$\begin{aligned} 5\,x_2 - 2\,x_3 &= -6, \\ 2\,x_1 + 4\,x_2 &= 7, \\ 3\,x_1 - 4\,x_3 &= 11 \end{aligned}$$

is $\begin{bmatrix} 0 & 5 & -2 & -6 \\ 2 & 4 & 0 & 7 \\ 3 & 0 & -4 & 11 \end{bmatrix}$.

(2) We may rewrite the above system in two ways:

$$\begin{bmatrix} 0 & 5 & -2 \\ 2 & 4 & 0 \\ 3 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -6 \\ 7 \\ 11 \end{bmatrix},$$

$$x_1 \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 5 \\ 4 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 0 \\ -4 \end{bmatrix} = \begin{bmatrix} -6 \\ 7 \\ 11 \end{bmatrix}.$$

[II] (4pts) (1) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + 2 \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}.$$

(2) $\begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} \begin{bmatrix} 2 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 2 \cdot 2 & 2 \cdot 5 & 2 \cdot 6 \\ 3 \cdot 2 & 3 \cdot 5 & 3 \cdot 6 \\ 0 \cdot 2 & 0 \cdot 5 & 0 \cdot 6 \end{bmatrix} = \begin{bmatrix} 4 & 10 & 12 \\ 6 & 15 & 18 \\ 0 & 0 & 0 \end{bmatrix}.$

[III] (8pts) (1) For $A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix}$,

(1)
$$\begin{aligned} AB &= \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 1 \cdot 2 + 2 \cdot 1 & 1 \cdot (-1) + 2 \cdot 4 \\ (-1) \cdot 2 + 1 \cdot 1 & (-1) \cdot (-1) + 1 \cdot 4 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 7 \\ -1 & 5 \end{bmatrix}. \end{aligned}$$

(2)
$$\begin{aligned} BA &= \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 \cdot 1 + (-1) \cdot (-1) & 2 \cdot 2 + (-1) \cdot 1 \\ 1 \cdot 1 + 4 \cdot (-1) & 1 \cdot 2 + 4 \cdot 1 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 3 \\ -3 & 6 \end{bmatrix}. \end{aligned}$$

(3) From (1), (2),
$$\begin{aligned} AB - BA &= \begin{bmatrix} 4 & 7 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 3 \\ -3 & 6 \end{bmatrix} \\ &= \begin{bmatrix} 4-3 & 7-3 \\ (-1)-(-3) & 5-6 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 4 \\ 2 & -1 \end{bmatrix}. \end{aligned}$$