Math 590 LINEAR ALGEBRA

MIDTERM EXAM I (Take-home)

April 5 (Tue), 2016

Due date: April 12 (Tue), 2016

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Line #: 52658.

ID #: ________________ Name: _______________________

This take-home part of Midterm Exam is worth 90 points and is due in class Tuesday, April 12th, 2016. Submission after 2:30 pm, April 12th, will not be accepted.

- Be sure to write your answers neatly, precisely, and with complete sentences. You may use notes and handed out materials, but no outside help.

[I] (Take-home; 30pts) For \( A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \) and \( B = \begin{bmatrix} p & q \\ r & s \end{bmatrix} \), prove

\[
\det(AB) = \left( \det A \right) \left( \det B \right).
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([I] continued)
(II) (Take-home; 30pts) For \( \mathbf{x} = \begin{bmatrix} p \\ q \end{bmatrix} \) and \( \mathbf{y} = \begin{bmatrix} r \\ s \end{bmatrix} \) define

\[
\mathbf{x} \ast \mathbf{y} = \begin{bmatrix} pr - qs \\ qr + ps \end{bmatrix}.
\]

(1) \( \square \quad \mathbf{x} \ast \mathbf{y} = \mathbf{y} \ast \mathbf{x} \).

\( \square \quad \mathbf{x} \ast \mathbf{y} \neq \mathbf{y} \ast \mathbf{x} \). \quad \text{(Check one.)}

(2) \[
\begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix} \ast \begin{bmatrix} \cos \phi \\ \sin \phi \end{bmatrix} = \begin{bmatrix} \cos(\square) \\ \sin(\square) \end{bmatrix} \quad \text{(explain)}. \]
Let \( A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \).

State **Cayley–Hamilton’s theorem** for \( A \). Then prove it.