Sample Math 115 Midterm Exam  
Fall, 2013

The midterm examination is on Wednesday, October 23, 5:45PM – 7:45PM.  
Note that you have to bring a #2 pencil to the exam.  
The problems below, all multiple-choice with exactly one correct answer, are intended to be reasonably representative of what might appear on the actual exam, which will have 25 problems.

1. The domain of function \( f(x) = \frac{x^3}{2x^2-x-3} \) is

   (A) \((-\infty, +\infty)\)  
   (B) \((-\infty, \frac{3}{2}) \cup \left(\frac{3}{2}, +\infty\right)\)  
   (C) \((-\infty, -1) \cup \left(\frac{3}{2}, +\infty\right)\)  
   (D) \((-\infty, -1) \cup (-1, +\infty)\)  
   (E) None of the above

2. Let \( f(x) = \frac{1}{x^2} \) and \( g(x) = 3x + 5 \). Then, \((g \circ f)(x)\) is

   (A) \(\frac{1}{(3x + 5)^2}\)  
   (B) \(\frac{3}{x^2} + 5\)  
   (C) \(\frac{1}{x^2}\)  
   (D) \(3x + 5\)  
   (E) None of the above

3. Let \( f(x) = x^2 + 1 \) and \( g(x) = \frac{1}{\sqrt{x}} \). Then, \( f(g(2)) \) is

   (A) \(\frac{1}{1 + \sqrt{2}}\)  
   (B) \(\frac{1}{\sqrt{5}}\)  
   (C) \(\frac{2}{3}\)  
   (D) \(\frac{3}{2}\)  
   (E) None of the above

4. Find the limit \( \lim_{x \to 3} \frac{x^2 - 9}{x - 3} \).

   (A) 1  
   (B) limit does not exist  
   (C) 6  
   (D) 3  
   (E) None of the above

5. Find the limit \( \lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x} \).

   (A) \(\frac{1}{2}\)  
   (B) limit does not exist  
   (C) 0  
   (D) \(-1\)  
   (E) None of the above

6. Find the limit \( \lim_{x \to \infty} \frac{3x^2 + 2x + 4}{2x^3 - 3x + 1} \).

   (A) 4  
   (B) \(\frac{3}{2}\)  
   (C) limit does not exist  
   (D) 0
7. Find the limit \( \lim_{x \to -\infty} \frac{x^2 + 3}{x + 1} \).

(A) 1   (B) 0   (C) 2   (D) 3

(E) None of the above

8. Let \( f(x) = x^2 - 4x \). When simplified, the difference quotient \( \frac{f(x+h) - f(x)}{h} \) becomes

(A) \( 2x + h - 4 \)   (B) 2   (C) 0   (D) \( 2x - 4 \)

(E) None of the above

9. Let \( f(x) = \frac{1}{x} \). When simplified, the difference quotient \( \frac{f(x+h) - f(x)}{h} \) becomes

(A) 1   (B) \( \frac{-1}{x(x+h)} \)   (C) \( \frac{1}{h^2} \)   (D) 0

(E) None of the above

10. The slope of the line through the points (-1,3) and (3,5) is

(A) \( \frac{1}{2} \)   (B) -2   (C) -4   (D) \( -\frac{1}{4} \)

(E) None of the above

11. What is the profit in terms of the cost \( C(x) \) to produce \( x \) units and the unit price \( p(x) \) at which \( x \) units will sell?

(A) \( x(p(x) - C(x)) \)   (B) \( p(x) - xC(x) \)   (C) \( p(x) + xC(x) \)   (D) \( xp(x) - C(x) \)

(E) None of the above

12. Given the demand equation \( 4x + 2p - 36 = 0 \) and the supply equation \( 2x - p + 10 = 0 \), where \( p \) is the unit price and \( x \) represents the quantity, find the equilibrium quantity and the equilibrium price.

(A) \( x = 2 \) and \( p = 14 \)   (B) \( x = 3 \) and \( p = 16 \)   (C) \( x = 4 \) and \( p = 18 \)

(D) \( x = 18 \) and \( p = 4 \)   (E) None of the above
13. It is known that \( \lim_{x \to 2^+} f(x) = 3, \lim_{x \to 2^-} f(x) = 3, \) and \( f(2) = 1. \) Which of the following statements is False?

(A) \( f(x) \) is discontinuous at \( x = 2. \)  
(B) The graph of \( f(x) \) is broken at \( x = 2. \)

(C) \( \lim_{x \to 2} f(x) \) exists.  
(D) \( f(x) \) is differentiable at \( x = 2. \)

(E) \( f(x) \) is defined at \( x = 2. \)

14. It is known that \( f(x) \) is continuous in \(( -\infty, \infty )\) and \( f(-1) = -2, f(0) = 2, \) and \( f(2) = 4. \) Which of the following statements is True?

(A) \( f(x) \) must have a zero in \((-1,0).\)  
(B) \( f(x) \) must have a zero in \((-2,-1).\)

(C) \( f(x) \) must have a zero in \((2,4).\)  
(D) \( f(x) \) must have a zero in \((0,2).\)

(E) None of the above is true.

15. An equation for the line tangent to the curve \( y = x^3 - 2x + 5 \) at the point \((-2, 1)\) is

(A) \( y + 1 = (2x^2 - 2)(x + 2) \)  
(B) \( y - 1 = (2x^2 - 2)(x + 2) \)  
(C) \( y = \frac{-x}{2} \)  
(D) \( y = 5x + 11 \)

(E) \( y = 10x + 21 \)

16. A ball is thrown straightly up into the air so that its height (in feet) after \( t \) seconds is given by \( s(t) = -16t^2 + 64t. \) The average velocity of the ball over the interval \([1, 1.05]\) is

(A) 49.56 ft/sec  
(B) 1.56 ft/sec  
(C) 31.2 ft/sec  
(D) 48 ft/sec  

(E) None of the above

17. Assume that the distance function \( s(t) \) is given as in Problem 16. The velocity of the ball at time \( t = 1 \) is

(A) 50 ft/sec  
(B) 2 ft/sec  
(C) 32 ft/sec  
(D) 48 ft/sec  

(E) None of the above

18. Find an equation of the tangent line at the point \((1, 32)\) of the graph of \( y = x(x+1)^5. \)

(A) \( y - 32 = (x + 1)^4(6x + 1) \)  
(B) \( y + 32 = (x + 1)^4(6x + 1) \)  
(C) \( y = 112x - 80 \)

(D) \( y = 112x + 80 \)  
(E) None of the above
19. For \( f(x) = \sqrt{2 + \sqrt{x}} \), evaluate \( f'(4) \).

\[
\begin{align*}
(A) & \quad \frac{1}{64} \\
(B) & \quad \frac{1}{16} \\
(C) & \quad \frac{1}{4} \\
(D) & \quad \frac{1}{2} \\
(E) & \quad \text{None of the above}
\end{align*}
\]

20. An equation for the line tangent to the curve \( y = (x^2 + x + 1)(x^3 - 2x + 2) \) at the point \((1, 3)\) is

\[
\begin{align*}
(A) & \quad y = 2x + 1 \\
(B) & \quad y = 3x \\
(C) & \quad y = 6x - 3 \\
(D) & \quad y = 7x - 4 \\
(E) & \quad \text{None of the above}
\end{align*}
\]

21. Let \( y = \sqrt{u} \) and \( u = 7x - 2x^2 \). Find \( \frac{du}{dx} \).

\[
\begin{align*}
(A) & \quad \frac{1}{2}(7 - 4x)(7x - 2x^2)^{1/2} \\
(B) & \quad \frac{1}{2}(7 - 4x)(7x - 2x^2)^{-1/2} \\
(C) & \quad \frac{1}{2}(7 - 4x) \\
(D) & \quad \frac{1}{2}(7x - 2x^2)^{-1/2} \\
(E) & \quad \text{None of the above}
\end{align*}
\]

22. Suppose that \( F(x) = f(x^2 + 1) \) and \( f'(2) = 3 \). Find \( F'(1) \).

\[
\begin{align*}
(A) & \quad 3 \\
(B) & \quad 4 \\
(C) & \quad 5 \\
(D) & \quad 6 \\
(E) & \quad \text{None of the above}
\end{align*}
\]

23. Suppose \( h = f \circ g \). Find \( h'(0) \) given that \( f(0) = 6, f'(5) = -2, g(0) = 5, \) and \( g'(0) = 3 \).

\[
\begin{align*}
(A) & \quad -6 \\
(B) & \quad 18 \\
(C) & \quad -10 \\
(D) & \quad 6 \\
(E) & \quad \text{None of the above}
\end{align*}
\]

24. What is the marginal profit in terms of the cost \( C(x) \) to produce \( x \) units and the unit price \( p(x) \) at which \( x \) units will sell ?

\[
\begin{align*}
(A) & \quad p'(x) - C'(x) \\
(B) & \quad p'(x) + xp(x) - C(x) \\
(C) & \quad x(p(x) - C(x)) \\
(D) & \quad p(x) + xp'(x) - C'(x) \\
(E) & \quad \text{None of the above}
\end{align*}
\]

25. Find \( \frac{dy}{dx} \) in terms of \( x \) and \( y \) when \( x \) and \( y \) are related by the equation \( x^2y - y^3 = 2 \).

\[
\begin{align*}
(A) & \quad \frac{xy}{3y^2 + x^2} \\
(B) & \quad \frac{x}{x - 3y} \\
(C) & \quad \frac{2xy}{3y^2 - x^2} \\
(D) & \quad \frac{x^2 + y^2}{2x + y} \\
(E) & \quad \text{None of the above}
\end{align*}
\]
26. Find \( \frac{dy}{dx} \) at point \((2, \sqrt{5})\) when \(x\) and \(y\) are related by the equation \(2x^2 - y^2 = 3\).

- (A) \( \frac{4}{\sqrt{5}} \)
- (B) \( \frac{2\sqrt{5}}{5} \)
- (C) \( \sqrt{5} \)
- (D) 2
- (E) None of the above

27. Find \( \frac{dy}{dx} \) in terms of \(x\) and \(y\) when \(x\) and \(y\) are related by the equation \(x^2 + y^2 + 2x^2y^2 = 10\).

- (A) \( -\frac{x}{y} \cdot (1 + 2y^2) \cdot x \cdot (1 + 2x^2) \)
- (B) \( -\frac{x}{y} \cdot (1 + 2x^2) \cdot y \cdot (1 + 2y^2) \)
- (C) \( -\frac{x}{y} \cdot (1 + 2x^2) \cdot y \cdot (1 + 2x^2) \)
- (D) \( -\frac{1}{x} \)
- (E) None of the above

28. The second derivative of function \( f(x) = (x^2 + 1)^5 \) is

- (A) \( 20(x^2 + 1)^3 \)
- (B) \( 10x(x^2 + 1)^4 \)
- (C) \( 10(x^2 + 1)^3(9x^2 + 1) \)
- (D) \( 10(x^2 + 1)^3(7x^2 + 4) \)
- (E) None of the above

29. The third derivative of \( f(x) = \frac{1}{x} \) is

- (A) \( -\frac{1}{x^2} \)
- (B) \( -\frac{2}{x^3} \)
- (C) \( \frac{2}{x^3} \)
- (D) \( \frac{1}{x^2} \)
- (E) None of the above

30. The distance \( s \) (in feet) covered by a car \( t \) seconds after starting from rest is given by \( s = -t^3 + 8t^2 + 20t \). Find the car’s acceleration at time \( t \).

- (A) \( -t^3 + 8t^2 + 20t \)
- (B) \( -3t^2 + 16t + 20 \)
- (C) \(-6t + 16 \)
- (D) \(-6t + 36 \)
- (E) None of the above

31. The domain of function \( f(x) = \frac{2x}{\sqrt{x^2 - 4}} \) is

- (A) \((2, +\infty)\)
- (B) \((-\infty, -2)\)
- (C) \((-\infty, -2] \cup [2, +\infty)\)
- (D) \((-\infty, -2) \cup (2, +\infty)\)
- (E) None of the above

32. Find the limit \( \lim_{x \to 0} \frac{(x+1)^2 - 1}{x} \).

- (A) 0
- (B) 1
- (C) 2
- (D) limit does not exist
- (E) None of the above

33. Suppose that \( h(x) = (2x + 1)g(x) \) and \( g(1) = -2, \ g'(1) = 3 \). Find \( h'(1) \).

- (A) 2
- (B) 3
- (C) 4
- (D) 5
- (E) None of the above
34. The average cost for manufacturing \( x \) units of a commodity is given by the function \( \bar{C}(x) \), and the demand equation is known to be \( p = f(x) \). Find the marginal profit.

(A) \( x \left( f(x) - \bar{C}(x) \right) \)  
(B) \( xf(x) - \bar{C}(x) \)  
(C) \( f(x) - \bar{C}(x) + x \left( f'(x) - \bar{C}'(x) \right) \)  
(D) \( f(x) + xf'(x) - \bar{C}'(x) \)  
(E) None of the above

35. It is known that \( f(x) \) is continuous in \((-\infty, \infty)\) and \( f(-1) = -4, f(0) = -2, \) and \( f(2) = 4 \). Which of the following statements is True?

(A) \( f(x) \) must have a zero in \((-1,0)\).  
(B) \( f(x) \) must have a zero in \((-2,-1)\).  
(C) \( f(x) \) must have a zero in \((2,4)\).  
(D) \( f(x) \) must have a zero in \((0,2)\).  
(E) None of the above is true.

36. Let \( f(x) = |x| \). Then,

(A) \( f \) is not defined at \( x = 0 \)  
(B) \( f \) has no limit at \( x = 0 \)  
(C) \( f \) is not continuous at \( x = 0 \)  
(D) \( f \) has no derivative at \( x = 0 \)  
(E) None of the above

37. The second derivative of function \( f(x) = (2x^2 + 1)^3 \) is

(A) \( 12x(2x^2 + 1)^2 \)  
(B) \( 6x(2x^2 + 1)^2 \)  
(C) \( 12(2x^2 + 1)(10x^2 + 1) \)  
(D) \( 6(2x^2 + 1)(8x^2 + 1) \)  
(E) None of the above

38. Let \( f(x) = \begin{cases} \sqrt{3x + 7} & \text{for } x < 3, \\ 10 - 2x & \text{for } x > 3. \end{cases} \)

(A) The domain of \( f \) is \((-\infty, +\infty)\)  
(B) \( \lim_{x \to 3^-} f(x) \) exists  
(C) \( f(x) \) is continuous at \( x = 3 \).  
(D) \( f(x) \) is differentiable at \( x = 3 \).  
(E) None of the above

39. Find the second-order derivative of function \( f(x) = \frac{x}{x^2 + 2} \).

(A) \( \frac{-4}{(x + 2)^2} \)  
(B) \( \frac{-4}{(x + 2)^3} \)  
(C) \( \frac{-4}{(x + 2)^2} \)  
(D) \( \frac{-4}{(x + 2)^4} \)  
(E) None of the above

40. Find \( \frac{dy}{dx} \) in terms of \( x \) and \( y \) when \( x \) and \( y \) are related by the equation \( x^{\frac{1}{3}} - y^{\frac{1}{3}} = 1 \).

(A) \( x^{-\frac{2}{3}}y^{-\frac{1}{3}} \)  
(B) \( x^{-\frac{2}{3}}y^{\frac{2}{3}} \)  
(C) \( x^{\frac{1}{3}}y^{-\frac{1}{3}} \)  
(D) \( x^{\frac{2}{3}}y^{\frac{2}{3}} \)  
(E) None of the above