Answer the following questions. Show detailed work. Answers without sufficient work will receive no credit.

1. (10 points each) In each of the following, find the function \( f(x) \) by finding the necessary antiderivatives.
   a. \( f'(x) = \frac{5x^2}{x^2+1}; \) and \( f(0) = 10. \)

   \[ f''(x) = \frac{3x^3 - 5x^2 + 1}{x^2}; \] and \( f(1) = 0, f(-1) = 1. \)

2. a. (5 points) Prove that \( \int_0^2 \sqrt{4-x^2} \, dx = \pi. \)

   b. (10 points) Use the midpoint rule with \( n = 4 \) to approximate the definite integral \( \int_0^2 \sqrt{4-x^2} \, dx. \)
2. Consider the function \( g(x) = \frac{5x}{x^2-1} \). Answer the following questions.
   a. (10 points) Find (with proof) all the horizontal and vertical asymptotes of the graph of \( y = g(x) \).

   b. (10 points) Find (with proof) all the local minimum and maximum values of \( y = g(x) \) if any exists.

   c. (10 points) Use the second derivative to determine the intervals on which the graph of \( y = g(x) \) is concave up or down.

3. (20 points) A sheet of metal measuring 6 ft by 12 ft is cut in two parts as shown in the figure below. The part on the left is then rolled horizontally while the part on the right is rolled vertically to make two drums. Find the value of \( x \) that yields the maximum possible total volume.
4. (5 points each) **Multiple choice:** In each of the following, circle only the one choice that would make the corresponding statement true. You do not have to show work for this question.

(i) The limit \( \lim_{x \to \infty} \left( 1 - \frac{20}{x} \right)^{5x} \)

(a) equals 0  
(b) equals 1  
(c) equals \( \frac{1}{e^{20}} \)  
(d) equals \( \frac{1}{e^{100}} \)

(ii) If the function \( y = f(x) \) is continuous on \( (-\infty, \infty) \), then

(a) it has no vertical asymptotes  
(b) it has exactly one vertical asymptote  
(c) it has to be differentiable  
(d) none of the above

(iii) If the function \( y = f(x) \) is continuous on its domain, \([-1, 1]\), then

(a) it has no extrema  
(b) it has both an absolute maximum and minimum  
(c) has to be differentiable on \((-1,1)\)  
(d) it has local but no absolute extrema

(iv) If the graph of the non-zero differentialble function \( y = f(x) \) is concave up, then the concavity of the graph of the function \( y = \frac{1}{f(x)} \)

(a) is always up  
(b) is always down  
(c) has an inflection point  
(d) depends on the functions \( f(x) \)

(v) If the non-zero differentialble function \( y = f(x) \) is always increasing, then the function \( y = \frac{1}{f(x)} \)

(a) is not differentiable  
(b) is always increasing  
(c) is always decreasing  
(d) none of the above