Math122 Review Problems for Midterm - Spring 2015
(Sec 8.1 - 8.8, H.1, H.2, 9.1 - 9.7)
Midterm Exam, Wednesday, March 11, 5:50 - 7:50 pm.

Review the Concept Check problems: Page 628/1 - 12, Page 688/1- 20

PART I: True-False Problems

Ch. 8. Page 629 True-False Quiz Problems 1 – 18.
Ch. 9. Page 688 True-False Quiz Problems 1 – 18.

Additional True-False Problems.

1. If the series \( \sum a_n \) converges, then the sequence \( \{a_n\} \) converges.
2. If \( \lim_{n \to \infty} |a_n| = 0 \), then \( \sum a_n \) converges.
3. If \( \sum a_n \) converges, then \( \sum |a_n| \) converges.
4. If \( \{a_n\} \) converges, then \( \{|a_n|\} \) converges.
5. If \( \{|a_n|\} \) converges, then \( \{a_n\} \) converges.
6. If \( \{a_n\} \) converges but \( \{b_n\} \) diverges, then \( \{a_n + b_n\} \) diverges.
7. If \( \{a_n\} \) converges but \( \{b_n\} \) diverges, then \( \{a_nb_n\} \) diverges.
8. If \( 0 \leq a_n \leq b_n \) for \( n \geq 1 \) and \( \{b_n\} \) converges, then \( \{a_n\} \) converges.
9. If \( 1 \leq a_n \leq b_n \) for \( n \geq 1 \) and \( \lim_{n \to \infty} b_n = 1 \), then \( \{a_n\} \) converges.
10. If \( \{a_n\} \) is bounded then \( \{a_n\} \) is convergent.
11. If \( 0 \leq a_n \leq b_n \) and \( \sum b_n \) converges, then \( \sum a_n \) converges.
12. If \( \sum a_n \) converges, then \( \lim_{n \to \infty} e^{a_n} = 1 \).
13. If \( \sum a_n \) is absolutely convergent, then \( \sum a_n^3 \) converges.
14. If \( f(x) = 1 - 3(x - 1)^2 + 5(x - 1)^3 + \cdots \) converges for all \( x \), then \( f''(1) = -3 \).
15. If \( \sum_{n=0}^{\infty} c_n(x - 1)^n \) is the Taylor series of \( f(x) = \ln x \) at \( a = 1 \), then \( c_2 = -1 \).
16. The points \( (1, \frac{4\pi}{3}) \) and \( (-1, \frac{\pi}{3}) \) represent the same point in the polar coordinate system.
17. For any vectors \( u \) and \( v \) in \( V_3 \), \( u \cdot v \) is a vector.
18. For any vectors \( u \) and \( v \) in \( V_3 \), \( u \times v \) is a vector.
19. For any vectors \( u, v \) and \( w \) in \( V_3 \), \( (u \times v) \cdot w \) is a vector.
20. If \(|\mathbf{u}|^2 + |\mathbf{v}|^2 = |\mathbf{u} + \mathbf{v}|^2\), then \(\mathbf{u} \cdot \mathbf{v} = 0\).

21. For any \(\mathbf{u} \in V_3\), \(\mathbf{u} \cdot \mathbf{u} = 0\).

22. For any \(\mathbf{u} \in V_3\), \(\mathbf{u} \times \mathbf{u} = 0\).

23. If two lines are perpendicular to a third line, then they are parallel.

24. If two lines are parallel to a third line, then they are parallel.

25. If two planes are perpendicular to a line, then they are parallel.

26. If two planes are parallel to a line, then they are parallel.

PART II: Multiple-Choice Problems

1. Exactly one of the following sequences diverges, Which is it?
   
   (A) \(\left\{ \frac{\sqrt{n^4 + 1}}{n^2} \right\}\)  
   (B) \(\left\{ \frac{n^2}{3^n} \right\}\)  
   (C) \(\left\{ \frac{2^n}{n!} \right\}\)  
   (D) \(\left\{ \frac{n}{(\ln n)^2} \right\}\)  
   (E) \(\left\{ \cos \frac{1}{\sqrt{n}} \right\}\)

2. Exactly one of the following sequences diverges, Which is it?
   
   (A) \(\left\{ \sin \frac{1}{\sqrt{n}} \right\}\)  
   (B) \(\left\{ \cos \frac{1}{\sqrt{n}} \right\}\)  
   (C) \(\left\{ n \sin \frac{1}{n} \right\}\)  
   (D) \(\left\{ e^{1/\sqrt{n}} \right\}\)  
   (E) \(\left\{ \frac{(-1)^n \sqrt{n} + 1}{\sqrt{n}} \right\}\)

3. Exactly one of the following series diverges, Which is it?
   
   (A) \(\sum_{n=1}^{\infty} \frac{1}{(-2)^n}\)  
   (B) \(\sum_{n=1}^{\infty} \frac{1}{n^4}\)  
   (C) \(\sum_{n=2}^{\infty} \frac{1}{n \ln n}^3\)  
   (D) \(\sum_{n=1}^{\infty} \frac{n^3 + n^2}{n^4 + 1}\)  
   (E) \(\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}\)

4. Exactly one of the following series diverges, Which is it?
   
   (A) \(\sum_{n=1}^{\infty} \frac{\sin n}{n^2}\)  
   (B) \(\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}\)  
   (C) \(\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}\)  
   (D) \(\sum_{n=2}^{\infty} \frac{\sin \frac{1}{n \ln n}}{n^5}\)  
   (E) \(\sum_{n=1}^{\infty} \frac{n^5}{n!}\)

5. The series \(\sum_{n=1}^{\infty} (-r)^n\) for \(0 < r < 1\) converges to
   
   (A) \(\frac{1}{1-r}\)  
   (B) \(\frac{1}{1+r}\)  
   (C) \(\frac{r}{1-r}\)  
   (D) \(\frac{r}{1+r}\)  
   (E) None of the above is true.

6. The sum of the geometric series \(\sum_{n=1}^{\infty} (-\pi)^{n-1}2^{-2n}\) is
   
   (A) \(\frac{1}{4-\pi}\)  
   (B) \(\frac{1}{4+\pi}\)  
   (C) \(-\frac{\pi}{4+\pi}\)  
   (D) \(\frac{1}{2+\pi}\)  
   (E) \(\frac{\pi}{4+\pi}\)
7. The series \( \sum_{n=1}^{\infty} \frac{1}{n(n + 1)} \) converges to

(A) \( \frac{1}{2} \)  \hskip 0.5cm (B) 1  \hskip 0.5cm (C) \( \frac{1}{3} \)  \hskip 0.5cm (D) \( \infty \)  \hskip 0.5cm (E) None of the above is true.

8. The sum of the series \( \sum_{n=1}^{\infty} \left( \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n + 2}} \right) \) is

(A) \( 1 + \frac{1}{\sqrt{3}} \)  \hskip 0.5cm (B) \( 1 - \frac{1}{\sqrt{3}} \)  \hskip 0.5cm (C) \( 1 + \frac{1}{\sqrt{2}} \)  \hskip 0.5cm (D) \( 1 - \frac{1}{\sqrt{2}} \)  \hskip 0.5cm (E) \( \infty \)

9. The sum of the series \( \sum_{n=0}^{\infty} \frac{1}{n!} \) is

(A) \( \sqrt{2} \)  \hskip 0.5cm (B) \( \pi \)  \hskip 0.5cm (C) \( e \)  \hskip 0.5cm (D) \( \ln 2 \)  \hskip 0.5cm (E) \( \infty \)

10. The sum of the series \( \sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n+1}}{(2n+1)!6^{2n+1}} \) is

(A) 0  \hskip 0.5cm (B) 1/2  \hskip 0.5cm (C) \( \pi/6 \)  \hskip 0.5cm (D) \( \sqrt{3}/2 \)  \hskip 0.5cm (E) \( \infty \)

11. The radius of convergence of the series \( \sum_{n=1}^{\infty} \frac{2 \ln n}{3n + 1} x^n \) is

(A) 1/3  \hskip 0.5cm (B) 2/3  \hskip 0.5cm (C) 1  \hskip 0.5cm (D) 0  \hskip 0.5cm (E) \( \infty \)

12. The interval of convergence of the series \( \sum_{n=1}^{\infty} \frac{3^n}{n} (x - 1)^n \) is

(A) \( [2/3, 4/3] \)  \hskip 0.5cm (B) \( (2/3, 4/3) \)  \hskip 0.5cm (C) \( (2/3, 4/3) \)  \hskip 0.5cm (D) \( [2/3, 4/3] \)  \hskip 0.5cm (E) None of the above is true.

13. The Maclaurin series for \( \cos(x^2) \) is

(A) \( \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n + 1} \)  \hskip 0.5cm (B) \( \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \)  \hskip 0.5cm (C) \( \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{(2n+1)!} \)

(D) \( \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \)  \hskip 0.5cm (E) \( \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{(2n)!} \)

14. The Maclaurin series for \( e^{-2x} \) is

(A) \( \sum_{n=0}^{\infty} (-1)^n \frac{2^n x^n}{n} \)  \hskip 0.5cm (B) \( \sum_{n=0}^{\infty} \frac{2^n x^n}{n!} \)  \hskip 0.5cm (C) \( \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{(2n)!} \)

(D) \( \sum_{n=0}^{\infty} (-1)^n \frac{2^n x^n}{n!} \)  \hskip 0.5cm (E) \( \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n!} \)
15. The 2nd degree Taylor polynomial of the function \( f(x) = \ln x \) at \( a = 1 \) is
(A) \( x - 1 - \frac{1}{2}(x-1)^2 \)  \( \quad \) (B) \( x - 1 - (x-1)^2 \)  \( \quad \) (C) \( x - x^2 \)  \( \quad \) (D) \( x - \frac{1}{2}x^2 \)  \( \quad \) (E) \( 1 - x \)

16. The 3rd degree Taylor polynomial of the function \( f(x) = \sin x \) at \( a = 0 \) is
(A) \( x \)  \( \quad \) (B) \( x \frac{x^3}{3!} \)  \( \quad \) (C) \( x \frac{x^3}{3!} + \frac{x^5}{5!} \)  \( \quad \) (D) \( 1 - \frac{x^2}{2!} \)  \( \quad \) (E) \( 1 - x - x^3 \)

17. The polar equation for the curve of the Cartesian equation \( x^2 + xy = 1 \) is
(A) \( \sin^2 \theta + \sin \theta \cos \theta = 0 \)  \( \quad \) (B) \( r (\sin^2 \theta + \sin \theta \cos \theta) = 1 \)  \( \quad \) (C) \( r^2 (\sin^2 \theta + \sin \theta \cos \theta) = 1 \)
(D) \( r (\cos^2 \theta + \sin \theta \cos \theta) = 1 \)  \( \quad \) (E) \( r^2 (\cos^2 \theta + \sin \theta \cos \theta) = 1 \)

18. The Cartesian equation of the polar equation \( r = \cos \theta - \sin \theta \) is
(A) \( y = x \)  \( \quad \) (B) \( y - x = 1 \)  \( \quad \) (C) \( x - y = 1 \)  \( \quad \) (D) \( x^2 + y^2 = \frac{1}{x - y} \)  \( \quad \) (E) \( x^2 + y^2 = \frac{1}{y - x} \)

19. The slope of the line tangent to the polar curve \( r = \sin \theta \) at \( \theta = \frac{\pi}{6} \) is
(A) \( \sqrt{3} \)  \( \quad \) (B) \( \frac{\sqrt{3}}{3} \)  \( \quad \) (C) \( -\sqrt{3} \)  \( \quad \) (D) \( -\frac{\sqrt{3}}{3} \)  \( \quad \) (E) \( \infty \)

20. The slope of the line tangent to the polar curve \( r = \theta \) at \( \theta = \pi \) is
(A) \( 0 \)  \( \quad \) (B) \( -1 \)  \( \quad \) (C) \( 1 \)  \( \quad \) (D) \( -\pi \)  \( \quad \) (E) \( \pi \)

21. The area of the region bounded by the polar curve \( r = \sqrt{\cos \theta} \) and the rays \( \theta = \frac{\pi}{6} \) and \( \theta = \frac{\pi}{2} \) is
(A) \( \frac{\pi}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{\cos \theta} d\theta \)  \( \quad \) (B) \( \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{\cos \theta} d\theta \)  \( \quad \) (C) \( \frac{\pi}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos \theta d\theta \)
(D) \( \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos \theta d\theta \)  \( \quad \) (E) \( \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\cos \theta)^2 d\theta \)

22. The exact length of the polar curve \( r = 1 + \cos \theta \) with \( \pi/6 \leq \theta \leq \pi/2 \) is
(A) \( \sqrt{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{1 + \cos \theta} d\theta \)  \( \quad \) (B) \( \frac{\sqrt{2}}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{1 + \cos \theta} d\theta \)  \( \quad \) (C) \( 2 \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1 + \cos \theta) d\theta \)
(D) \( \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1 + \cos \theta) d\theta \)  \( \quad \) (E) \( 4 \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1 + \cos \theta)^2 d\theta \)

23. If \(|\mathbf{a}| = 1\), \(|\mathbf{b}| = 2\), and the angle between \( \mathbf{a} \) and \( \mathbf{b} \) is \( \pi/3 \), then \(|\mathbf{a} - \mathbf{b}| =
(A) \sqrt{5} \quad (B) 5 \quad (C) \sqrt{3} \quad (D) \sqrt{5 - 2\sqrt{3}} \quad (E) 3 \)
24. If \(|a| = 1\), \(|b| = 2\), and \(|a \times b| = \sqrt{3}\), then the angle between \(a\) and \(b\) is

(A) either 0 or \(\pi\)   (B) either \(\pi/6\) or \(5\pi/6\)   (C) either \(\pi/3\) or \(2\pi/3\)   (D) either \(\pi/4\) or \(3\pi/4\)   (E) \(\pi/2\)

25. The scalar projection \(\text{comp}_u v\) of \(u = \langle 1, -1, \sqrt{2} \rangle\) onto \(v = \langle 0, -3, 4 \rangle\) is

(A) \(3 + 4\sqrt{2}\)   (B) \(\frac{3 + 4\sqrt{2}}{2}\)   (C) \(\frac{3 + 4\sqrt{2}}{4}\)   (D) \(\frac{3 + 4\sqrt{2}}{24}\)   (E) \(\frac{3 + 4\sqrt{2}}{5}\)

26. The vector projection \(\text{proj}_u v\) of \(v = \langle 0, -3, 4 \rangle\) onto \(u = \langle 1, -1, \sqrt{2} \rangle\) is

(A) \(\frac{3 + 4\sqrt{2}}{2}\)   (B) \(\frac{3 + 4\sqrt{2}}{2}\langle 1, -1, \sqrt{2} \rangle\)   (C) \(\frac{3 + 4\sqrt{2}}{4}\langle 1, -1, \sqrt{2} \rangle\)   (D) \(\frac{3 + 4\sqrt{2}}{5}\langle 1, -1, \sqrt{2} \rangle\)   (E) \(\frac{3 + 4\sqrt{2}}{25}\langle 0, -3, 4 \rangle\)

27. The area of the triangle with vertices at the points \(P(1, 2, 3)\), \(Q(-1, 0, 1)\) and \(R(1, 1, 0)\) is

(A) \(\sqrt{14}\)   (B) \(2\sqrt{14}\)   (C) \(3\sqrt{14}\)   (D) \(4\sqrt{14}\)   (E) \(5\sqrt{14}\)

28. The volume of the parallelepiped determined by the vectors \(a = \langle 1, 2, 3 \rangle\), \(b = \langle -1, 0, 1 \rangle\) and \(c = \langle 1, 1, 0 \rangle\) is

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

29. The distance from the point \(P(1, 2, 3)\) to the line through the points \(Q(-1, 0, 1)\) and \(R(1, 1, 0)\) is

(A) \(\frac{\sqrt{42}}{6}\)   (B) \(\frac{\sqrt{42}}{3}\)   (C) \(\frac{2\sqrt{42}}{3}\)   (D) \(\frac{2\sqrt{21}}{3}\)   (E) \(\sqrt{14}\)

30. The distance from the point \(P(1, 2, 3)\) to the line \(r(t) = \langle 0, 1, 1 \rangle + t\langle -2, 1, 2 \rangle\) is

(A) 5   (B) \(\sqrt{5}\)   (C) \(\sqrt{15}\)   (D) 0   (E) 1

31. The distance from the point \(P(1, 2, 3)\) to the plane through the points \(Q(1, 1, 3)\), \(R(4, 1, 0)\), and \(S(-1, -1, 3)\) is

(A) 1   (B) \(\frac{2}{\sqrt{3}}\)   (C) \(\frac{1}{\sqrt{3}}\)   (D) \(\frac{5}{\sqrt{3}}\)   (E) \(\frac{1}{3}\)

32. The distance from the point \(P(1, 2, 3)\) to the plane \(x - y + z = 3\) is

(A) 1   (B) \(\frac{2}{\sqrt{3}}\)   (C) \(\frac{1}{\sqrt{3}}\)   (D) \(\frac{5}{\sqrt{3}}\)   (E) \(\frac{1}{3}\)
33. If \( \theta \) is the angle between the planes \( x - y + z = 3 \) and \( 2x - y - z = 1 \), then \( \cos \theta \) is

(A) \( \sqrt{2} \)  (B) \( \frac{\sqrt{2}}{2} \)  (C) \( \frac{3\sqrt{2}}{2} \)  (D) \( \frac{\sqrt{2}}{3} \)  (E) \( \frac{\sqrt{2}}{6} \)

34. The distance between the planes \( x - 3y + 2z = 3 \) and \( 2x - 6y + 4z = 3 \) is

(A) \( \frac{1}{2\sqrt{14}} \)  (B) \( \frac{1}{\sqrt{14}} \)  (C) \( \frac{3}{2\sqrt{14}} \)  (D) \( \frac{2}{\sqrt{14}} \)  (E) \( \frac{5}{2\sqrt{14}} \)

35. Exactly one of the following vectors is parallel to the line described by \( x = 1 + 2t, y = -3t, z = 3 - t \). Which is it?

(A) \( \langle 1, 0, 3 \rangle \)  (B) \( \langle -1, 0, -3 \rangle \)  (C) \( \mathbf{i} - 3\mathbf{j} + 4\mathbf{k} \)  (D) \( -2\mathbf{i} + 3\mathbf{j} + \mathbf{k} \)  (E) \( \langle 2, 0, -1 \rangle \)

36. Exactly one of the following vectors is normal to the plane described by \( 4x - 6y = 5 - 2z \). Which is it?

(A) \( \langle 2, -3, 1 \rangle \)  (B) \( 4\mathbf{i} - 6\mathbf{j} - 2\mathbf{k} \)  (C) \( 2\mathbf{i} - 3\mathbf{j} + \mathbf{k} \)  (D) \( 4\mathbf{i} + 6\mathbf{j} + 2\mathbf{k} \)  (E) \( \langle 4, -6, -5 \rangle \)

37. The domain of the function \( f(x, y) = \ln\left(\frac{x - y^2}{\sqrt{1 - x}}\right) \) is

(A) \( \{(x, y) | x \geq y^2\} \)  (B) \( \{(x, y) | y^2 < x < 1\} \)  (C) \( \{(x, y) | y^2 < x, x \geq 1\} \)

(D) \( \{(x, y) | y^2 < x \leq 1\} \)  (E) \( \{(x, y) | y^2 \leq x < 1\} \)

38. Exactly one of the following equations in cylindrical coordinates completely describes the cone \( x^2 + y^2 - z^2 = 0 \). Which is it?

(A) \( r = z \)  (B) \( r = -z \)  (C) \( r = z^2 \)  (D) \( r^2 = z^2 \)  (E) \( r^2 \cos 2\theta = z^2 \)

39. Exactly one of the following equations in spherical coordinates completely describes the ellipsoid \( 2x^2 + 5y^2 + 2z^2 = 1 \). Which is it?

(A) \( \rho^2(2 + 3\sin^2 \phi \sin^2 \theta) = 1 \)  (B) \( \rho^2(2 + 3\cos^2 \phi \sin^2 \theta) = 1 \)  (C) \( \rho^2(2 + 3\sin^2 \phi \cos^2 \theta) = 1 \)

(D) \( \rho^2(2 + 3\cos^2 \phi \cos^2 \theta) = 1 \)  (E) \( \rho^2(2 + 3\cos^2 \phi) = 1 \)

PART III. Essay Problems

1. Determine whether the sequence converges. Find the limit if it is convergent.

(a) \( a_n = \frac{n^2 - n}{2n^2 - 3} \)  (b) \( a_n = \frac{\ln(2n + 1)}{n} \)  (c) \( a_n = \frac{2^n}{e^{n+2}} \)

(d) \( a_n = \frac{n \cos n}{e^n} \)  (e) \( a_n = \ln(n^2 + 1) - \ln(2n^2 - n) \)  (f) \( a_n = (1 + 2/n)^{2n} \)
2. Let the sequence \( \{a_n\} \) satisfy
\[
a_1 = 10, \quad a_{n+1} = \sqrt{6 + a_n}, \quad n \geq 1.
\]
(a) (optional) Show that \( a_n > 0 \) and \( a_n \geq a_{n+1} \) for \( n \geq 1 \). (Hence, \( \{a_n\} \) has a limit since it is decreasing and bounded below.)
(b) Find the limit.

3. Determine whether the series is convergent, or divergent, or absolutely convergent.

\[
\begin{align*}
(a) & \sum_{n=1}^{\infty} \frac{\sin^n n}{2^n} & (b) & \sum_{n=1}^{\infty} \frac{2^n \cos(n\pi)}{e^n} & (c) & \sum_{n=1}^{\infty} \frac{(-1)^n (\ln n)^2}{n} \\
(d) & \sum_{n=1}^{\infty} \frac{(-1)^n n^2 + e^{-n}}{2n^2 + 1} & (e) & \sum_{n=1}^{\infty} \frac{(-1)^{n-1} n!}{1 \cdot 3 \cdot 5 \cdots (2n + 1)} & (f) & \sum_{n=1}^{\infty} \frac{(n + 1)^2}{n^3 (-3)^n}
\end{align*}
\]

4. Use the partial sum \( s_5 \) to estimate the sum of the series \( \sum_{n=1}^{\infty} \frac{1}{n^3} \), and estimate the error using \( s_5 \) as an approximation of the sum of the series.

5. Use the partial sum \( s_5 \) to estimate the sum of the series \( \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3} \), and estimate the error using \( s_5 \) as an approximation of the sum of the series.

6. Find the radius of convergence and interval of convergence of the power series.
\[
\begin{align*}
(a) & \sum_{n=0}^{\infty} \frac{(1-x)^n}{2n + 1} & (b) & \sum_{n=1}^{\infty} \frac{(n+2)^n}{n^2 3^n} & (c) & \sum_{n=1}^{\infty} \frac{x^n}{n^2 3^n} & (d) & \sum_{n=1}^{\infty} \frac{3^{n+1} (x-3)^n}{n!}
\end{align*}
\]

7. Find the Taylor series of the function at \( a = 0 \).
\[
\begin{align*}
(a) & \frac{1}{1-x^2} & (b) & \frac{1}{(1-x)^2} & (c) & \ln(1+x) & (d) & \sin(x^2) & (e) & x^2 e^{-x}
\end{align*}
\]

8. Find the Maclaurin series of \( e^{-x^2} \) and approximate \( \int_0^{0.1} e^{-x^2} \, dx \) correct to within an error of \( 10^{-5} \).

9. Find the Taylor polynomial \( T_3(x) \) of the function \( e^{\sin x} \) at \( a = 0 \).

10. Let \( T_3(x) \) be the degree 3 Taylor polynomial of \( e^{x^2} \) at \( a = 0 \). Use the Taylor inequality to find a bound for
\[
|R_3(x)| = \left| e^{x^2} - T_3(x) \right|
\]
for \( x \in [0, 0.1] \).
11. Find the Cartesian coordinates of the point given in polar coordinates.
   (a) \( (2, \frac{\pi}{6}) \)  
   (b) \( (4, \frac{3\pi}{4}) \)  
   (c) \( (0, \frac{\pi}{5}) \)  
   (d) \( (5, -\frac{\pi}{2}) \)  
   (e) \( (3, -\frac{\pi}{3}) \)

12. Find the polar coordinates \((r, \theta)\) with \( r \geq 0 \) and \( 0 \leq \theta < 2\pi \) of the point given in Cartesian coordinates
   (a) \((1, 0)\)  
   (b) \((3, \sqrt{3})\)  
   (c) \((-2, 2)\)  
   (d) \((-1, \sqrt{3})\)  
   (e) \((0, -2)\)

13. Find the slope of the line tangent to the polar curve at the point specified by the value of \( \theta \).
   (a) \( r = \sin \theta + \cos \theta, \quad \theta = \frac{\pi}{6} \)  
   (b) \( r = 1 + \theta^2, \quad \theta = \frac{\pi}{2} \)  
   (c) \( r = 4 \cos 3\theta, \quad \theta = \frac{\pi}{6} \)

14. Find the points on the given curve where the tangent line is horizontal or vertical.
   (a) \( r = 1 + \cos \theta \)  
   (b) \( r^2 = \cos 2\theta \)

15. Find the area of the region that is bounded by the given curve and lies in the specified sector.
   (a) \( r = 1 - \cos \theta, \quad 0 \leq \theta \leq \frac{\pi}{2} \)  
   (b) \( r = 3 - \theta, \quad 0 \leq \theta \leq 3 \)

16. Find the area of the region enclosed by one loop of the curve.
   (a) \( r = 2 \sin \theta \)  
   (b) \( r^2 = \cos 2\theta \)

17. Find the area of the region that lies inside \( r = \sqrt{2} \cos \theta \) and outside \( r = 1 \).

18. Find the exact length of the polar curve.
   (a) \( r = \theta^2, \quad 0 \leq \theta \leq \pi \)  
   (b) \( r = 2 \cos \theta, \quad 0 \leq \theta \leq \frac{\pi}{2} \)

19. Let \( \mathbf{a} \) and \( \mathbf{b} \) be vectors in \( V_2 \). Suppose \( |\mathbf{a}| = 1, |\mathbf{b}| = 2 \), and the angle between \( \mathbf{a} \) and \( \mathbf{b} \) is \( \theta = \pi/4 \). Find
   (a) \( \mathbf{a} \cdot \mathbf{b} \)  
   (b) \( |\mathbf{a} + 2\mathbf{b}| \)  
   (c) \( |3\mathbf{a} - 2\mathbf{b}| \)  
   (d) \( |(2\mathbf{a}) \times \mathbf{b}| \)  
   (e) \( \text{comp}_a \mathbf{b} \)  
   (f) \( \text{comp}_b \mathbf{a} \)  
   (d) The area of the parallelogram determined by \( \mathbf{a} \) and \( \mathbf{b} \).

20. Suppose that \( \mathbf{a} = \langle 1, -1, 2 \rangle \), \( \mathbf{b} = -\mathbf{i} + 3\mathbf{k} \) and \( \mathbf{c} = \langle -2, 3, 1 \rangle \). Find
   (a) \( \mathbf{a} \cdot (\mathbf{b} - 2\mathbf{c}) \)  
   (b) \( (2\mathbf{a} - \mathbf{c}) \times \mathbf{b} \)  
   (c) \( \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) \)  
   (d) \( \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) \)  
   (e) The angle between \( \mathbf{a} \) and \( \mathbf{b} \)
   (f) The area of the parallelogram determined by \( \mathbf{a} \) and \( \mathbf{b} \)
   (g) The volume of the parallelepiped determined by \( \mathbf{a}, \mathbf{b} \) and \( \mathbf{c} \)
21. Given the points \( P(1, 3, -1), Q(2, -1, 1), R(1, 1, 1), \) and \( S(-2, 1, -3) \), find

(a) The area of the triangle with vertices \( P, Q, R \)
(b) The length of the line segment \( PQ \)
(c) The angle between the vectors \( \overrightarrow{PQ} \) and \( \overrightarrow{PR} \).
(d) Parametric equations for the line that passes through the points \( Q \) and \( R \)
(e) The distance form the point \( P \) to the line that passes through \( Q \) and \( R \)
(f) A scalar equation for the plane through the points \( Q, R, \) and \( S \)
(g) The distance from the point \( P \) to the plane through the points \( Q, R, \) and \( S \)
(h) The volume of the parallelepiped with adjacent edges \( PQ, PR, \) and \( PS \)
(i) The distance between the line through the points \( P, Q \) and the line through the points \( R \) and \( S \)
(j) Symmetric equations for the line through the point \( P \) and normal to the plane through the points \( P, R \) and \( S \)
(k) The angle between the plane through the points \( P, Q, R \) and the plane through the points \( P, Q, S \)

22. Find the value(s) of \( t \) such that the vectors \( \langle t, 2t - 1, 3 \rangle \) and \( \langle t + 1, -1, -1 \rangle \) are orthogonal.

23. Find the work done by a force \( \mathbf{F} = 2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k} \) that moves a particle from the point \( P(1, 1, 1) \) to the point \( Q(4, -1, 2) \).

24. Find the work done by a force of 10N applied at an angle of \( \pi/6 \) to the moving direction in moving an object 3m.

25. Find the line (its vector equation, parametric equations, and symmetric equations) that passes through the point \( P(1, 2, 3) \) and is perpendicular to the plane \( 2x - y + 3z = 9 \).

26. Find the line (its vector equation, parametric equations, and symmetric equations) that passes through the points \( P(1, 2, 3) \) and \( Q(3, 2, 1) \).

27. Find the line (its vector equation, parametric equations, and symmetric equations) that passes through the point \( P(1, 1, 1) \) and is perpendicular to the vectors \( \langle 1, 1, 0 \rangle \) and \( \langle 1, 0, 1 \rangle \).

28. Find the line (its vector equation, parametric equations, and symmetric equations) of the intersection of the planes \( z = x + y - 1 \) and \( y = x + z + 1 \).

29. Find a scalar equation of the plane through the point \( P(1, 0, 2) \) with normal vector \( \mathbf{n} = \langle 2, -1, 3 \rangle \).
30. Find a scalar equation of the plane that passes through the point \( P(1, 2, 3) \) and is perpendicular to the line \( 2(x - 1) = \frac{y}{2} = -\frac{z - 5}{4} \).

31. Find a scalar equation of the plane that contains the line \( 2(x - 1) = \frac{y}{2} = -\frac{z - 5}{4} \) and is parallel to the vector \( \langle 1, 1, 1 \rangle \).

32. Find a scalar equation of the plane that passes through the point \( P(1, 1, 1) \) and is parallel to the vectors \( \langle 1, 1, 0 \rangle \) and \( \langle 1, 0, 1 \rangle \).

33. Show that the lines
\[
L_1 : \{ x = 1 + t, y = -2 - t, z = 3t \} \quad \text{and} \quad L_2 : \{ x = 2 - 3t, y = 1 - t, z = 3 + 3t \}
\]
are skew. Find the plane that contains the line \( L_1 \) and is parallel to the line \( L_2 \). Determine the distance between \( L_1 \) and \( L_2 \).

34. Find (a) the angle, (b) the distance between the two planes \( x + 3y = z + 2 \) and \( z = x + y - 1 \).

35. Let \( f(x, y) = \frac{\sqrt{xy - 1}}{x^2 - 4} \).
   (a) Find the domain of \( f(x, y) \). (b) Evaluate \( f(5, 2) \).

36. Let \( f(x, y) = \sqrt{4 - y^2} \ln(y^2 - x) \).
   (a) Find the domain of \( f(x, y) \). (b) Evaluate \( f(0, -1) \).

37. The cylindrical coordinates of a point are \( (2\sqrt{3}, \pi/3, 2) \). Find the rectangular and spherical coordinates of the point.

38. The rectangular coordinates of a point are \( (2, 2, -1) \). Find the cylindrical and spherical coordinates of the point.

39. The spherical coordinates of a point are \( (8, \pi/4, \pi/6) \). Find the rectangular and cylindrical coordinates of the point.

40. Write the equation in cylindrical coordinates and in spherical coordinates.
   (A) \( x^2 + y^2 + z^2 = 4 \) (B) \( x^2 + z^2 = 4 \).