Math 122 Sample Midterm - Spring 2015
(This sample exam is used to show the format of Midterm only! It is NOT an indication of the types of problems that will appear in the actual Midterm!)

Instructions
1. You will be given 2 hours for this exam.
2. The exam has three parts: True-False, Multiple-Choice, and Essay Problems.
3. Fill in your name, Student ID, and your instructors name at the top of this page.
4. Write your name in the upper left corner of each new page.
5. You may detach the blank sheet at the end, but not other sheets.
6. Only TI-83/84 type calculators (or similar) are allowed. Books and notes are not allowed.
7. Copy the letters of your answers to True-False and Multiple-Choice Problems in the space below. Make sure you copied them correctly.
8. No partial credit will be received for True-False and Multiple-Choice Problems.
9. For Essay Problems do all your work in this booklet, circle your final answers, and write them in the appropriate boxes.
10. Show your work neatly for Essay Problems. Unjustified or illegible correct answers may receive no credit. Partial credit may be received based on your work.
11. You are not allowed to borrow or interchange calculators during the exam.
12. Turn off your cell phone and any other electronic devices (but calculators) and keep them in your pocket or bag.

Part A: True-False Problem Answers.

________________________ correct (5 points each) Total ___________

Part B: Multiple-Choice Problem Answers.

17. _____ 18. _____
________________________ correct (7 points each) Total ___________

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<tr>
<th>True or False (1-8)</th>
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Part A. True-False Problems

For problems 1 through 8, circle “T” if you think the statement is always true; Otherwise, circle “F”. Copy your answers to the cover sheet.

1. If \( \lim_{n \to \infty} a_n = 0 \), then \( \sum_{n=0}^{\infty} a_n \) converges. \[ T \quad F \]

2. If \( \{a_n\} \) converges, then \( \{|a_n|\} \) converges. \[ T \quad F \]

3. If \( \sum_{n=0}^{\infty} a_n \) converges, then \( \sum_{n=0}^{\infty} |a_n| \) converges too. \[ T \quad F \]

4. If \( \sum_{n=0}^{\infty} a_n 4^n \) converges, then \( \sum_{n=0}^{\infty} a_n (-4)^n \) converges. \[ T \quad F \]

5. In \( \{a_n\} \) converges but \( \{b_n\} \) diverges, then \( \{a_n + b_n\} \) diverges. \[ T \quad F \]

6. For any 3-D vectors \( \mathbf{u} \) and \( \mathbf{v} \), both \( \mathbf{u} \cdot \mathbf{v} \) and \( \mathbf{u} \times \mathbf{v} \) are vectors. \[ T \quad F \]

7. If two lines are perpendicular to a third line, then they are parallel. \[ T \quad F \]

8. If two planes are perpendicular to a line, then they are parallel. \[ T \quad F \]
Part B. Multiple-Choice Problems

For problems 9 through 18, circle only one answer that you think is correct. Copy your answers to the cover sheet.

9. Exactly one of the following sequences diverges. Which is it?
   \[
   (A) \left\{ \frac{n}{2^n} \right\}_{n=1}^\infty \quad (B) \left\{ \frac{\sqrt{n^2 + n + 1}}{n} \right\}_{n=1}^\infty \quad (C) \left\{ \frac{1 - \cos n}{n} \right\}_{n=1}^\infty \quad (D) \left\{ \frac{(-1)^n n}{\ln n} \right\}_{n=2}^\infty \quad (E) \left\{ \sin \frac{1}{n} \right\}_{n=1}^\infty
   \]

10. Exactly one of the following series diverges. Which is it?
    \[
    (A) \sum_{n=1}^\infty \frac{1}{3^n} \quad (B) \sum_{n=1}^\infty \frac{n^2}{n^3 + 1} \quad (C) \sum_{n=1}^\infty \frac{(-1)^n}{n} \quad (D) \sum_{n=1}^\infty \frac{1}{n^3} \quad (E) \sum_{n=1}^\infty \frac{n}{n^3 + 1}
    \]

11. The interval of convergence of the power series \( \sum_{n=5}^\infty \frac{2^n x^n}{n} \) is
    \[
    (A) (-2, 2) \quad (B) [-2, 2) \quad (C) (-2, 2] \quad (D) (-0.5, 0.5) \quad (E) [-0.5, 0.5)
    \]

12. Let \( \theta \) be the angle between the planes \( x - 2y + 2z = \sqrt{2} \) and \( x + y + z = 0 \). Then \( \cos \theta \) equals
    \[
    (A) \frac{1}{\sqrt{3}} \quad (B) \frac{1}{\sqrt{3}} \quad (C) \frac{1}{3\sqrt{3}} \quad (A) \frac{1}{3\sqrt{3}} \quad (A) \frac{3}{\sqrt{3}}
    \]

13. The Cartesian equation of the polar equation \( r = \frac{1}{r - \cos \theta} \) is
    \[
    (A) x^2 + y^2 = x \quad (B) x^2 + y^2 = y \quad (C) x^2 + y^2 - x = 1 \quad (D) x^2 + y^2 - y = 1 \quad (E) x^2 + y^2 = \frac{1}{x^2 + y^2 - x}
    \]
14. Let $\mathbf{a} = \langle -1, -2, 2 \rangle$ and $\mathbf{b} = \langle 3, 0, 4 \rangle$ be two vectors. The vector projection $\text{proj}_b \mathbf{a}$ of $\mathbf{a}$ onto $\mathbf{b}$ is

\begin{align*}
(A) & \frac{1}{5} & (B) & \langle -1, 2, 2 \rangle & (C) & \left\langle \frac{-3}{5}, 0, \frac{-4}{5} \right\rangle & (D) & \left\langle \frac{-5}{9}, \frac{-10}{9}, \frac{10}{9} \right\rangle & (E) & \left\langle \frac{3}{5}, 0, \frac{4}{5} \right\rangle
\end{align*}

15. The volume of the parallelepiped whose adjacent edges are the line segments from $(0, 0, 0)$ to the points $(1, 1, -1), (1, -2, 1)$ and $(1, 1, 1)$ is

\begin{align*}
(A) & 0 & (B) & 2 & (C) & 4 & (D) & 6 & (E) & 8
\end{align*}

16. The Taylor series of $f(x) = \frac{x^2}{(1-x)^2}$ at $a = 0$ is

\begin{align*}
(A) & \sum_{n=0}^{\infty} x^n & (B) & \sum_{n=1}^{\infty} x^n & (C) & \sum_{n=1}^{\infty} nx^n & (D) & \sum_{n=2}^{\infty} (n-1)x^n & (E) & \sum_{n=3}^{\infty} (n-2)x^n
\end{align*}

17. Exactly one of the following vectors is normal to the plane described by $4x - 6y = 5 - 2z$. Which is it?

\begin{align*}
(A) & \langle 2, -3, 1 \rangle & (B) & 4i - 6j - 2k & (C) & 2i - 3j - k & (D) & 4i + 6j + 2k & (E) & \langle 4, -6, -5 \rangle
\end{align*}

18. The domain $D$ of the function $f(x, y) = \frac{\ln(x - y^2)}{\sqrt{1-x}}$ is

\begin{align*}
(A) & \{(x, y) | x \geq y^2 \} & (B) & \{(x, y) | y^2 < x < 1 \} & (C) & \{(x, y) | x > y^2, x \geq 1 \} \\
(D) & \{(x, y) | y^2 < x \leq 1 \} & (E) & \{(x, y) | y^2 \leq x < 1 \}
\end{align*}
Part C . Essay Problems
For problems 19 through 21, show your work to get credit.

19. (30 pts)
(A) Find the Taylor series of \( f(x) = \frac{1}{1+x^4} \) at \( a = 0 \).

Answer:

(B) Find a power series of \( \int \frac{1}{1+x^4} \, dx \) by using (A), and determine the radius of convergence and the interval of convergence of the obtained power series.

Answer:

(C) Use the Taylor series obtained in (B) too approximate the integral \( \int_0^{0.25} \frac{1}{1+x^4} \, dx \) correct within \( 10^{-5} \).

Answer:
20. (30 pts) Let \( \mathbf{a} \) and \( \mathbf{b} \) vectors in \( V_3 \). Suppose that \( |\mathbf{a}| = 4 \), \( |\mathbf{b}| = 2 \), and the angle between \( \mathbf{a} \) and \( \mathbf{b} \) is \( \theta = \pi/3 \).

(A) Determine \( |\mathbf{a} - 2\mathbf{b}| \).

(B) Find the area of the parallelogram determined by \( \mathbf{a} \) and \( \mathbf{b} \).
21. (30 pts)

(A) Determine the domain of the function \( f(x, y) = \frac{\sqrt{x - y^2}}{\ln(y^2 - 1)}. \)

Answer:

(B) Write the equation \( x^2 + y^2 + z^2 = 4 \) in cylindrical coordinates.

Answer:

(C) Find the area of the region that is bounded by the curve \( r = 1 - \cos \theta \) and lies in the sector \( 0 \leq \theta \leq \frac{\pi}{2} \).

Answer: