Math 122 - Quiz 1

NAME: __________________________ LAB: 1PM 2PM (circle one)

1. (2 points) Find a formula for the general term $a_n$ of the sequence \( \left\{ \frac{1}{4}, \frac{\text{-}2}{9}, \frac{3}{16}, \frac{-4}{25}, \ldots \right\} \) assuming the pattern of the first few terms continues.

\[
\begin{align*}
a_1 &= \frac{1}{4} = \frac{1}{2^2} \\
a_2 &= -\frac{2}{9} = -\frac{2}{3^2} \\
a_3 &= \frac{3}{16} = \frac{3}{4^2} \\
&\vdots \\
a_n &= (-1)^{n+1} \frac{n}{(n+1)^2}
\end{align*}
\]

2. (2 points) Is the following sequence monotonic? Is it bounded?

\( \left\{ \frac{1}{2n+3} \right\}_{n=1}^\infty \)

Note that for \( f(x) = \frac{1}{2x+3} \),

\[
\begin{align*}
f'(x) &= -\frac{2}{(2x+3)^2} < 0 \quad \text{for all } x \geq 1 \\
\end{align*}
\]

So, \( \left\{ \frac{1}{2n+3} \right\} \) is decreasing (i.e. monotonic.)

Since it's decreasing, it is bounded above.

Furthermore, \( \frac{2n+3}{1} > 0 \) for all \( n \geq 1 \).

I.e. it's bounded below.

Hence, the sequence is bounded.
3. (2 points each) Determine whether each of the following sequences converge or diverge. If it converges, find its limit.

(a) \( \{ \cos(2/n) \}_{n=1}^{\infty} \)

\[
\text{Since } \frac{2}{n} \to 0 \text{ as } n \to \infty \text{ and } f(x) = \cos(x) \text{ is continuous}
\]

\[
l_n \lim_{n \to \infty} \cos \left( \frac{2}{n} \right) = \cos \left( \lim_{n \to \infty} \frac{2}{n} \right) = \cos(0) = 1.
\]

(b) \( \left\{ \frac{3^{n+2}}{5^n} \right\}_{n=1}^{\infty} \)

\[
\frac{3^{n+2}}{5^n} = 3 \cdot \left( \frac{3}{5} \right)^n \to 0 \text{ as } n \to \infty \text{ since } \left| \frac{3}{5} \right| < 1.
\]

(c) \( \{ \sin \left( \frac{n}{5000\pi} \right) \}_{n=1}^{\infty} \)

\[
\text{Since } \frac{n}{5000\pi} \to \infty \text{ as } n \to \infty \text{ and sine oscillates on these values, this sequence diverges.}
\]