**Math 115 - LIMITS**

**Definition:** A function $f$ has limit $L$ as $x$ approaches $a$, written as $\lim_{x \to a} f(x) = L$, means as $x$ gets close to $a$, but not equal $a$, $f(x)$ gets close to $L$.

**Properties of Limits:** Suppose $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$

1. $\lim_{x \to a} c = c \quad (c \in \mathbb{R})$
2. $\lim_{x \to a} x = a$
3. $\lim_{x \to a} (f(x))^r = (\lim_{x \to a} f(x))^r = L^r \quad (r \in \mathbb{R})$
4. $\lim_{x \to a} cf(x) = c(\lim_{x \to a} f(x)) = cL$
5. $\lim_{x \to a} (f(x) \pm g(x)) = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x) = L \pm M$
6. $\lim_{x \to a} f(x)g(x) = (\lim_{x \to a} f(x))(\lim_{x \to a} g(x)) = LM$
7. $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{L}{M}$ if $M \neq 0$

**Definition:** $\lim_{x \to \infty} f(x) = L$ means as $x$ gets larger and larger, $f(x)$ gets close to $L$. 

$\lim_{x \to -\infty} f(x) = L$ means as $x$ gets smaller and smaller, $f(x)$ gets close to $L$.

**Fact:** For $n > 0$, $\lim_{x \to \infty} \frac{1}{x^n} = 0$ and $\lim_{x \to -\infty} \frac{1}{x^n} = 0$.

**Definition:** $\lim_{x \to a^-} f(x) = L$ means as $x$ approaches $a$ from the left side of $a$, but not equal $a$, $f(x)$ gets close to $L$. $\lim_{x \to a^+} f(x) = L$ means as $x$ approaches $a$ from the right side of $a$, but not equal $a$, $f(x)$ gets close to $L$.

**Fact:** $\lim_{x \to a} f(x) = L$ iff $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = L$

**Definition:** A function $f$ is **continuous** at $x = a$ if

1. $f(a)$ is defined,
2. $\lim_{x \to a} f(x) = L$, and
3. $L = f(a)$.

**Fact:** If a function $f$ is the quotient of two polynomials $p(x), q(x)$, $f(x) = \frac{p(x)}{q(x)}$, then $f$ is continuous at all $x = a$ such that $Q(a) \neq 0$. 

IVT: (Intermediate Value Theorem) Let $f$ be a function defined on a closed interval $[a, b]$. If $w$ is between $f(a)$ and $f(b)$, there is at least one $c \in [a, b]$ such that $f(c) = w$. 