DEPARTMENT OF MATHEMATICS
UNIVERSITY OF KANSAS
MATH 647 - Fall 2011 - Midterm

Your Name: ________________________________

On this exam, you may use a calculator and the book, as well as a page of notes.

It is not sufficient to just write down the answers. You must explain how you arrived at your answers and how you know they are correct.

1 (60) _______
2 (60) _______
3 (60) _______
4 (60) _______
5 (60) _______
Total (300) _______
(1) Find the sine series expansion of the function $f(x) = x(1 - x)$ on $0 < x < 1$. 
(2) Compute \( \int_{-\pi}^{\pi} f^2(x)dx \) for the function \( f(x) = 1 + \sum_{n=1}^{\infty} \left( \frac{\cos(nx)}{2^n} + \frac{\sin(nx)}{n^2} \right) \) using Parseval’s identity.
(3) Solve the one dimensional wave equation with \( c = 1/\pi \) for a string of unit length with fixed ends and initial conditions

\[
\begin{align*}
u(x, 0) &= x \sin \pi x, \\
\frac{\partial u}{\partial t}(x, 0) &= 0.
\end{align*}
\]

Hint: Use the trigonometric identity

\[
\sin mx \sin nx = \frac{1}{2} [\cos (m - n)x - \cos (m + n)x].
\]
(4) Use D’Alembert’s formula to solve the one dimensional wave equation with $c = 1$ boundary value problem for a string of unit length with fixed ends, subject to $u(x, 0) = 0, \frac{\partial u}{\partial t}(x, 0) = -10$. Describe completely the function $G$ (antiderivative of $g^*$).
(5) Solve the non-homogeneous boundary value problem for the one dimensional heat equation on a bar with unit length and $c = 1$, for the following data:

$$u(0, t) = 100, \ u(1, t) = 50, \ u(x, 0) = x.$$