MATH 950, FALL 2013

Homework Set II:

• Problem 1
  Find the solution of \((x + 1)^2 u_x + (y - 1)^2 u_y = (x + y) u\) satisfying the condition \(u(x, 0) = -1 - x\) for \(-1 < x < \infty\). Where in the \(xy\)-plane is \(u(x, y)\) determined by these conditions?

• Problem 2
  Show that all the projected characteristic curves of
  \[(2x + u)u_x + (2y + u)u_y = u\]
  through the point \((1, 1)\) are given by the straight line \(y = x\).

• Problem 3
  Solve \(xu_x + yu_y + (u_x^2 + u_y^2)/2 = u\) with initial condition \(u(x, 0) = (1 - x^2)/2\).

• Problem 4
  Solve the equation \((u_x)^2 + (u_y)^2 = 1\) with initial data given by \(s \rightarrow (\sin s, \cos s, 0)\) for \(0 \leq s \leq \pi/2\). Based on the method of characteristics where in the \(xy\)-plane is \(u(x, y)\) determined by these conditions?

• Problem 5
  Read the application to geometrical optics section on p.36 - 40 in McOwen’s book and then work out problem 9 on page 42.