Final Homework Set:

• **Problem 1** Use separation of variables to find a solution of the PDE

\[ tu_t = u_{xx} + 2u, \quad u(0, t) = u(\pi, t) = 0 \]

Show that there are an infinite number of solutions that satisfy the initial condition \( u(x, 0) = 0 \) and thus uniqueness is false for this equation.

• **Problem 2** Find solitary wave solutions of the Boussinesq equation

\[ u_{tt} - u_{xx} + 3(u^2)_{xx} - u_{xxxx} = 0 \]

in the form \( u(x, t) = a\text{sech}^2 b(x - ct) \) and show that a wave may propagate in either direction.

• **Problem 3** Solve \( u_x^2 + yu_y - u = 0 \) with initial condition

\[ u(x, 1) = \frac{x^2}{4} + 1. \]

• **Problem 4** Find the explicit formula for the traveling wave solution

\[ u(x, t) = f(x - ct) \]

of the Korteweg-deVries (KdV) equation

\[ u_t + u_{xxx} + 6uu_x = 0, \quad -\infty < x < \infty \]

• **Problem 5** Problem 1, section 3.2 of McOwen’s book, page 90

• **Problem 6** Problem 6, section 3.3 of McOwen’s book, page 94

• **Problem 7** Problem 6, section 4.4 of McOwen’s book, page 137

• **Problem 8** Problem 4, section 5.3 of McOwen’s book, page 158