DEPARTMENT OF MATHEMATICS
UNIVERSITY OF KANSAS
MATH 290 - Fall 2005 - EXAM 2

Your Name: ________________________________

On this exam, you may use a calculator, but no books or notes.

It is not sufficient to just write down the answers. You must explain how you arrived at your answers and how you know they are correct.

1 (40) ________
2 (40) ________
3 (40) ________
4 (40) ________
5 (40) ________
6 (20) ________
Total (200) ________
(1) (a) Find the area of a triangle $ABC$, where $A = (2, -1)$, $B = (-4, 8)$ and $C = (2, -13)$.

(b) Find the equation of the line, passing through $A$ and $C$. 
(2) If \( A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{pmatrix} \) and \( B = \begin{pmatrix} 1 & 2 & 1 \\ 0 & k & 1 \\ 0 & -1 & k \end{pmatrix} \) for some parameter \( k \), determine \( \det(AB) \), \( \det(A^T B) \), \( \det(AB^2 A) \).
(3) (a) Verify whether \( v_1 = (1, 1, -2), v_2 = (2, 5, -1), v_3 = (0, 1, 1) \) is a basis for \( \mathbb{R}^3 \).

(b) Find the rank of the matrix
\[
A = \begin{pmatrix}
1 & 1 & 1 & 2 & 0 \\
-5 & 3 & 3 & -10 & -8 \\
1 & 2 & -3 & 2 & 14 \\
-1 & 1 & 1 & -2 & -2
\end{pmatrix}
\]

(c) Find a basis for the row space of \( A \).
(4) (a) Find the standard matrix for $T : \mathbb{R}^3 \to \mathbb{R}^2$, defined by $T(x, y, z) = (x - y, x + y + z)$.
(b) Find the standard matrix for the linear transformation that corresponds to a reflection with respect to the line $y = -x$. 
\textbf{Hint: For b) compute first } $T(1, 0)$ \textbf{ and } $T(0, 1)$ \textbf{ and take it from there.}
(5) Find a basis for the solution space of $Ax = 0$, where $A =
\begin{pmatrix}
1 & 1 & 0 \\
-2 & -2 & 0 & 0 \\
0 & 0 & 1 & 1 \\
-1 & -1 & 0 & 0 \\
\end{pmatrix}$
(6) (Bonus problem 20 points) Define the linear transformation “integration” $T : P_2 \to P_3$, that is $T(1) = x, T(x) = x^2/2, T(x^2) = x^3/3$.
(a) Write the standard matrix for $T$.
(b) Show that if $D$ is the linear transformation “differentiation”, then $D \circ T$ is the identity transformation on $P_2$. Is the same true for $T \circ D$? Why?