(1) Problem 19/page 23;
(2) Problem 29/page 24;
(3) Problem 34/page 25;
(4) Problem 47/page 26;
(5) Show that the functions
\[ f(x, y) = \frac{y}{x^2 + y^2}; \quad g(x, y) = -\frac{x}{x^2 + y^2} \]
satisfy \( f_y = g_x \) for each \( \mathbb{R}^2 \setminus \{0\} \), but on the other hand there is no \( C^2 \) function \( h \) on \( \{(x, y) : 0 < x^2 + y^2 < 1\} \) so that
\[ h_x = f, \quad h_y = g. \]
Explain why this does not contradict the generalized version of Theorem 1.5.1 that we have established in class.
**Hint:** To show the non-existence of \( h \) argue by contradiction, by considering the path integral
\[ \int_{x^2 + y^2 = 1} f(x, y)dx + g(x, y)dy. \]
(6) Problem 55/page 27 without the counterexample. I will discuss the counterexample later.