1. (a) Compute the line integral of the vector field \( \mathbf{F} = (3x^2y, x^3 + 3y^2) \) along the segment from (1, 1) to (2, 2) by direct computation.

(b) Show that \( \mathbf{F} = (3x^2y, x^3 + 3y^2) \) is a conservative vector field.

(c) Find a potential for \( \mathbf{F} = (3x^2y, x^3 + 3y^2) \).

(d) Compute the line integral of the vector field \( \mathbf{F} = (3x^2y, x^3 + 3y^2) \) along any curve from (1, 1) to (2, 2).

2. (a) Consider the vector field \( \mathbf{G}(x, y) = (2y + x^3, x) \). Show that \( \mathbf{G} \) is not conservative.

(b) Compute the line integral \( \oint_C \mathbf{G} \cdot d\mathbf{x} \), where the curve \( C \) is the boundary of the square \([0, 1] \times [0, 1]\) oriented counterclockwise.

3. Consider the curve \( \gamma \) given by the three sides of the triangle from (0, 0) to (1, 0) to (0, 1) oriented counterclockwise. Show that

\[
\int_\gamma (-xy + \sin x^2) \, dx + \cos y^2 \, dy = 1/6.
\]

4. Show that the line integral of \( \mathbf{F} = (x/(x^2 + y^2), y/(x^2 + y^2)) \) along any closed curve that does not have the point (0, 0) in its interior is zero.

5. Let \( \mathbf{F} \) and \( \mathbf{G} \) are vector fields in \( \mathbb{R}^3 \) (appropriately differentiable). Show that

\[
\text{div}(\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}.
\]

6. Compute the flux of the vector field \( \mathbf{F}(x, y, z) = (x, y, z) \) across the sphere of radius one in the direction to the normal pointing to the inside.

7. Let \( S \) be the parametric surface given by

\[
\mathbf{X}(x, z) = (x, x^3 + z, z),
\]

for \( 0 \leq x \leq 2 \) and \( 0 \leq z \leq 3 \).

(a) Find the equation of the tangent plane to surface \( S \) at the point (1,2,1).

(b) Set up an integral to compute the area of the parametric surface \( S \). DO NOT COMPUTE THE INTEGRAL.

8. (a) Compute the unit normal vector \( \mathbf{n} \) pointing to the outside at each point of the cylinder given by

\[
\mathbf{X}(u, v) = (\cos u, \sin u, v)
\]

for \( 0 \leq u \leq 2\pi, 0 \leq v \leq 1 \).
(b) Compute the flux of the vector field $\mathbf{F}$ across the cylinder $S$ of part (a),

$$\text{FLUX} = \int \int_S \mathbf{F} \cdot d\mathbf{S},$$

where

$$\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}.$$

9. Compute

$$\int \int_S z^3 dS$$

where $S$ is the sphere of radius one centered at the origin.

10. Consider the paraboloid $M$ given by $z = 1 - (x^2 + y^2)$ for $0 \leq x^2 + y^2 \leq 1$.

(a) Write a parametrization of $M$ of the form $\mathbf{f}(x, y) = (x, y, h(x, y))$.

(b) Compute the unit normal to the surface $M$, "pointing up".