A brief guideline for Math 648 Exam 1 SP2016

• Exam 1 is scheduled on Feb. 25. The exam coverage is up to Section 4.

• Basics: You are expected to remember (if you don’t want to re-derive) the First Variation and Euler-Lagrange equation for each of the following forms of optimization problems:

  (i) \( J(y) = \int_{x_0}^{x_1} f(x, y, y') dx \) with \( y(x_0) = y_0, \ y(x_1) = y_1; \)

  (ii) \( J(y) = \int_{x_0}^{x_1} f(x, y, y'') dx \) with \( y(x_0) = y_0, \ y'(x_0) = p_0, \ y(x_1) = y_1, \ y'(x_1) = p_1; \)

  (iii) \( J(q) = \int_{t_0}^{t_1} f(t, q(t), q'(t)) dt \) with \( q(t_0) = q_0, \ q(t_1) = q_1; \)

  (iv) \( J(y) = \int_{x_0}^{x_1} f(x, y, y') dx \) (or \( J(q) = \int_{t_0}^{t_1} f(t, q, \dot{q}) dt \))

  with fixed boundary conditions and subject to the isoperimetric constraint \( I(y) = \int_{x_0}^{x_1} g(x, y, y') dx = L \) (or \( I(q) = \int_{t_0}^{t_1} g(t, q, \dot{q}) dt = L \)).

• Special conserved quantities such as \( H = y' f_y' - f \) if \( f(x, y, y') = f(y, y') \) for special cases of \( f \) are important for solving Euler-Lagrange equations.

• Techniques: In addition to everything from calculus, you are expected to know how to solve

  – First order linear, separable, and exact ODEs;
  – Higher order linear ODEs with constant coefficients (homogeneous and nonhomogeneous);
  – Linear systems of first order ODEs with constant coefficients.