Math 648 S16 Homework 6 (Due Tuesday 05/03/16)

• Section 9.3, page 220: #2.

**Hint:** The equation for the generator $\xi$ and $\eta$ in (9.38) yields that

$$
\xi_y = \eta_x = 0, \quad \xi + x\eta_y - \frac{1}{2} x\xi_x = 0, \quad \frac{1}{3} y^6 \xi + xy^5 \eta + \frac{1}{6} xy^6 \xi_x = 0.
$$

$\xi_y = \eta_x = 0$ gives that $\xi$ is a function of $x$ only and $\eta = \eta(y)$ is a function of $y$ only. The equation $\xi + x\eta_y - \frac{1}{2} x\xi_x = 0$ then implies that $\eta_y$ is a function of $x$, which gives that $\eta(y) = ay + b$ for some constant $a$.

Now substitute $\eta(y) = ay + b$ into the last two equations above to get $b = 0$ and

$$
x\xi_x - \xi = 0.
$$

The latter 1st order linear ODE gives $\xi = cx$ for some constant $c$. Then substitute $\xi = cx$ to either of the two equations to get $\eta = -\frac{c}{3} y$.

A first integral is then given by

$$
x^2 \left( yy' + x (y'^2 + \frac{1}{3} y^6) \right).
$$

*(The answer in the book seems wrong.)*

• Section 8.2, page 170: #1. The Lagrangian should be

$$
L(t, q, \dot{q}) = \frac{1}{2} m \dot{q}^2 - \frac{1}{2} k q^2.
$$

• Section 8.4 (optional), page 184: #1, #2, #3.